WITHDRAWABLE BIDS AS WINNER’S CURSE INSURANCE

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When bidders have a common value or strongly affiliated values for an object or contract being auctioned by sealed bids, it is possible that the maker of a rational and apparently winning bid would, upon learning the competing bids, prefer losing the auction to honoring his bid. The ability to withdraw a bid, perhaps at a cost, in such circumstances provides a form of “winner’s curse insurance.” Bidding with such insurance is analyzed, obtaining the general condition for rational bid withdrawal and sufficient conditions for the existence of an equilibrium with more aggressive bidding. Next, we define a “compensation penalties” bid withdrawal penalty scheme and show that with it the bid-taker is better off on the average than if bid withdrawal is impossible. Finally, we find equilibrium bidding and withdrawal strategies in a multiplicative model as a function of the magnitude of the bid withdrawal penalty and of the bid-taker’s likelihood, after a withdrawal, of honoring the second-best bid. This model has cases in which allowing withdrawal at a cost is in the bid-taker’s interest and ones in which it is not.

Most bidding theory treats submitted bids as firm enforceable commitments. In some situations, this is literally correct, and in others a bidder would have little or no reason for wanting to withdraw a winning bid. However, winning bids are sometimes withdrawn. These bid withdrawals can occur for various reasons. They may be motivated by

1. a mistake in the original bid;
2. a desire to have the second-best bid honored;
3. unexpected events beyond the winning bidder’s control (such as the refusal of a bank to issue a mortgage);
4. information obtained after the auction but before completion of the transaction (such as a fall in prices or the failure of similar assets to sell in subsequent auctions);
5. an opportunistic victimization of an unwary seller after the competing bidders are no longer available; or
6. discouraging information about the expected profitability of winning the auction inferred from the rivals’ bids.

This paper deals with the last of these motivations. In the context of common values or affiliated values, there are situations in which the information contained in the bids of competitors can be sufficiently surprising and distressing to the winning bidder that he would wish to withdraw his rationally chosen winning bid even if the withdrawal incurs a penalty. The ability to withdraw an otherwise winning bid in these circumstances can serve as valuable “insurance” against the winner’s curse. This value may also provide the bid-taker with incentive to announce the losing bids when he would not otherwise do so.

Consider a low-bid-wins auction, say a competition for a construction contract. In an independent-private-values context, a winning bidder would learn nothing about his cost of doing the job from the losing bids. However, in an uncertain situation involving strictly affiliated costs, a winning bidder may learn something about his own costs from competitors’ bids. Suppose that a bidder makes an unbiased estimate of his cost based upon his private information without any adjustment for his rivals’ private information. Upon learning the magnitudes of his rivals’ bids, the winner can re-estimate his cost. Given that he is the winner, this re-estimate will be higher. There is a possibility that it will be so much higher that he would profit from withdrawing his winning bid and losing the auction even if a penalty, pecuniary or nonpecuniary, was attached to the withdrawal. The optimal criterion for withdrawal for a bidder maximizing expected gain is essentially that the bidder’s revised estimate of the cost of the job exceed his “winning” bid by more than the withdrawal penalty.

In the face of such a withdrawn bid, the buyer may choose to contract with the next lowest bidder or he may refuse to buy from any of the bidders. In the latter case, it is clear that when there is a positive probability of a bid withdrawal, the presence of this possibility should lead to more aggressive bidding. In essence, the reason for this is that the presence of the withdrawal opportunity increases the expected value of winning the contract because its only effect is to allow the “winner” to avoid some potential losses. However, if there is some possibility that the buyer will award the contract to the next highest bidder should a “winning” bid be withdrawn, then there are offsetting forces and the possibility of less aggressive bidding cannot be ruled out, in general, by use of this argument.

Modeling withdrawable bids may be a helpful step toward bridging the chasm between formal models of pure

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auctions and some of the complex competitive situations in which auctions play a role.

In the first modeling of withdrawable bids of which we are aware, Rothkopf (1991) presented various models of auctions in which bidders can submit multiple bids and, perhaps, for a fee, withdraw a winning bid to win with his own less aggressive one. None of these models, however, allows for the possibility that a bidder will withdraw an otherwise winning bid to lose an auction. This possibility is analyzed here.

Two other papers offer game-theoretic models of auction situations in which a winning bid may be withdrawn in order to lose. In each case, the ex post information is statistically exogenous, unlike the information contained in rivals’ bids. Von Ungern-Sternberg (1991) considers bidders in a construction contract auction whose only uncertainty regarding their cost of contract fulfillment is over a capacity-stretching cost that arises if they should win both the auction analyzed and another auction; these events of winning are uncorrelated. If the option of withdrawing a winning bid is available and avoids capacity-stretching costs, a bidder may bid more aggressively. Waehrer (1991) considers sale by auction of an asset for which a public estimate of asset value will become known after bids are submitted but before completion of the transaction. Each bidder’s value is a function of this common post-auction estimate and of his own private information, an independent random variable. It may be in the seller’s interest to offer a withdrawal option at a relatively low penalty.

Spulber (1990) considers the related issue of price modification in response to cost overruns during the performance of a contract awarded in an auction.

The rest of the paper has five sections. The first of these specifies the conditions under which a rational bidder would wish to withdraw an otherwise winning bid. Section 2 presents sufficient conditions for the existence of an equilibrium in which the presence of the right to withdraw bids results in more aggressive bidding. Section 3 presents an analysis of auctions conducted under “compensation penalties,” an economically focal bid withdrawal penalty scheme. With this scheme, the bidding is always more aggressive and the bid-taker is always better off than in an auction without the possibility of bid withdrawal. The next section presents a specific game-theoretic model in which the bidders follow contingent bid withdrawal policies. For its equilibrium, we obtain the strategies, the expected bidder profit, the probability that no transaction takes place and the bid-taker’s net cost. The final section discusses our results.

1. THE OPTIMALITY CONDITION FOR BID WITHDRAWAL

We now present a formal statement of the optimal condition for the withdrawal of the lowest bid. A completely analogous condition holds for the withdrawal of the highest bid when the bidders are competing to buy.

Consider an auction for the supply of a service with n bidders participating, each trying to maximize noncooperatively his expected profit from this one auction. Assume that for j = 1, 2, ..., n, bidder j has an unknown cost C_j of providing the service and that he receives a signal X_j (e.g., a cost estimate) and then follows a strategy of using a strictly increasing function of this signal b_j(X_j) to determine his only bid, B_j. Denote the inverse function of b_j by b_j^-1. If bidder i’s bid of B_i is the lowest bid, then he faces a decision as to whether to go through with the contract to provide the services or to incur an immediate penalty that he evaluates as p_i(B_i). Under these circumstances, it is optimal for bidder i to withdraw his bid if

B_i + p_i(B_i) ≤ E[C_j|X_j = b_j^{-1}(B_j), j = 1, 2, ..., n] \text{ (1)}

and optimal not to withdraw it if the reverse inequality holds.

The introduction of bidder risk aversion would seriously complicate our calculations, perhaps intractably, and detract from the message of this paper. However, we note in passing that risk-averse bidders are likely to be particularly helped by the “winner’s curse insurance” offered by the possibility of withdrawing an otherwise winning bid in circumstances in which new information indicates that there is a sizable probability of a large loss. A bid-taker might benefit more by offering bid withdrawal possibilities to bidders willing to pay a risk premium than by doing so to risk-neutral bidders.

2. SUFFICIENT CONDITIONS FOR MORE AGGRESSIVE BIDDING

As mentioned, the ability to withdraw, at a finite cost, an otherwise winning bid offers a type of “winner’s curse insurance.” What conditions suffice to guarantee that this insurance leads to more aggressive bidding than would occur if the withdrawal cost were infinite (i.e., withdrawal was impossible)? Recall that the value of the right to withdraw a bid is positive whenever there is a positive probability that a bidder would prefer to withdraw; hence, given such a positive probability, one sufficient condition is that no contract will be awarded in the event of a withdrawal.

A potentially countervailing incentive arises when there is some chance that a bidder will profit from a contract awarded to him because a rival who underbid him withdraws. When this is possible, less aggressive bidding may conceivably be equilibrium behavior.

To develop another sufficient condition for more aggressive bidding, consider a symmetric, n bidder, lowest-bid-wins model with symmetric equilibria in which the bidders’ costs are strictly affiliated. To reduce complications, assume for this section that there is a fixed penalty p ≥ 0 for withdrawing a bid, an exogenous probability α
that the withdrawal of the lowest bid will result in an award to the second-lowest bidder, and no chance at all that the withdrawal of the second-lowest bid will result in the award to any other bidder.

Suppose that there is a unique symmetric equilibrium in which each bidder uses an increasing bid function $B(x)$, and follows the optimality condition for bid withdrawal presented in the previous section. Let $b(x)$ be the symmetric equilibrium bid function for the standard first-price auction, i.e., the auction with no possible bid withdrawal. We will say that the withdrawal option has led to more aggressive bidding if $B(x) < b(x)$ for all $x$ and $B(x) < b(x)$ for some $x$.

Let bidder 1 observe cost estimate $X_1 = x$ and consider the impact on his expected profit of an incremental change in his bid, $d\beta$, relative to a reference bid $\beta$, when he has the option to withdraw at penalty $p$. One part of the impact of an incrementally more aggressive bid is that bidder 1 will submit the lowest bid slightly more often. There is an overall calculation of the expected profitability conditional on $\beta + d\beta$ becoming the lowest bid; we extract from this calculation only the portion that is due to the opportunity for bid withdrawal limiting the size of possible losses and call this part of the impact $I_a(x, \beta, p)$.

A second part of the impact is that an incrementally higher bid will lead a rival who submitted the lowest bid to incrementally overestimate bidder 1’s cost estimate $X_1$; if bidder 1 submitted the second-lowest bid, there is some incremental probability that the incrementally higher bid will lead to bidder 1 being awarded a contract after withdrawal that he would otherwise have lost. Label this second impact $I_b(x, \beta, p, \alpha)$; it reflects a small probability that an incremental change in 1’s bid could switch the lowest bidder’s withdrawal decision, but a possibly high profit in this event. This occurs because bidder 1’s rational bid placed a high probability on the event that all rivals had higher cost estimates than his, and so his estimate of the contract fulfillment cost will be lower if a rival with a lower cost estimate than his withdraws.

In addition to the two impacts just labeled, there are other ways in which the withdrawal possibility alters expected profitability calculations. Some of these are second-order impacts and, hence, can be neglected. Others cancel out of an incremental analysis of bidding aggressiveness, such as the expected profitability of a bid withdrawal that would yield a profit to bidder 1 whether he bid $\beta$ or $\beta + d\beta$. Hence, a sufficient condition for greater bidding aggressiveness is that this latter impact not influence marginal profitability as much as the former, that is,

$$I_a(x, \beta, p) > I_b(x, \beta, p, \alpha).$$

The Appendix provides precise definitions of these impacts, proofs of sufficiency, and these three results:

1. Equilibrium bidding is more aggressive with withdrawal for sufficiently small probabilities, $\alpha$, of an award to the second lowest bidder in the event of a bid withdrawal;

2. Equilibrium bidding is more aggressive with withdrawal if the number of bidders, $n$, is large enough; and

3. For many distributions of costs and estimates, equilibrium bidding is more aggressive with withdrawal if the variability of the estimating distribution is sufficiently large.

3. AUCTIONING UNDER COMPENSATION PENALTIES FOR BID WITHDRAWAL

This section considers auctions with bid withdrawals subject to “compensation penalties.” The basic idea of a compensation penalty is that the bid represents a firm offer, but the damages for withdrawal for which the bidder is liable are limited to the damages suffered by the bid-taker because of the withdrawal. Thus, if a bid-taker receives offers to do a job for $200, $300, and $700, the low bidder is liable for a bid withdrawal penalty of $100 ($= $300 – $200) if he withdraws his bid. If he does withdraw it, then the bid offering to do the work for $300 may be awarded the contract. If this bidder also withdraws, his penalty is $400 ($= $700 – $300). Finally, we note that the highest bidder may not withdraw his bid under this penalty system, but the issue should never arise if the bids are chosen rationally to begin with and the only new information available to the highest bidder is the amounts of the competing bids.

We will consider a class of affiliated value models with compensation penalties. However, to get a simplified version of the results to come, consider first a low-bid-wins common-value auction model with compensation penalties for bid withdrawal and a bid-taker who always awards the contract to the next lowest bidder if the lowest remaining bid is withdrawn. As in other standard game-theoretic models of common-value auctions, each bidder can use the publicly opened bids of all bidders to calculate an updated estimate of the common cost, and each bidder will arrive at the same estimate. Under these circumstances, if one or more bidders have made bids below this common post-auction estimate, all but the highest of these will withdraw their bids. The maker of the highest of these bids will accept the contract because the penalty for withdrawing his bid is greater than his expected loss from carrying out the contract. The bidder undertaking the contract has a positive expected profit if and only if he bid above the common post-auction estimate; this cannot occur when one or more rivals bidding lower withdrew. With two bidders, there will be no bid withdrawals. With three or more bidders and with nonpathological estimating distributions, there
will be in equilibrium a positive probability of a bid withdrawal, and, hence, there will be more aggressive bidding than in the corresponding no-withdrawal-allowed auction.\(^8\)

Next, we consider more formally a class of affiliated values auctions with compensation penalties. Let \(C_0\) have continuous distribution function \(F(c_0)\). Given any realization \(c_0 \sim C_0\), let each bidder \(i = 1, \ldots, n\) observe a cost estimate \(X_i\), with the \(X_i\) identically and conditionally independently distributed as \(G(x_i|c_0)\). For ease of exposition, assume \(G\) to be continuous and unbiased. If bidder \(i\) is offered the contract after bids are submitted (either initially or following a rival’s withdrawal), his cost of contract fulfillment is \(C_i = \lambda C_0 + (1 - \lambda)X_i\), where \(\lambda\) is a known parameter. Thus, \(\lambda, 0 \leq \lambda \leq 1\), represents the degree of commonality of costs across bidders: \(\lambda = 1\) gives the common-value model, while \(\lambda = 0\) gives a private-values model in which no bid withdrawals occur. In what follows, we will assume neither of these extreme cases applies.

Assume that some increasing function \(b(x)\) is a symmetric equilibrium bid function for this model with compensation penalties for bid withdrawal, and assume that \(\alpha = 1\), i.e., the buyer always approaches the next bidder if a bidder withdraws. As a bidder’s expected profit must be nonnegative, unbiased estimates imply \(b(x) > x\). To analyze bid withdrawal and expected profit considerations, assume the vector of bids \((B_1, \ldots, B_n)\) has been submitted. Without loss of generality, bidders can be assumed to be numbered so that the vector of bids is arrayed ascendingly. As the bid function \(b(x)\) is increasing, announcement of the bids allows all bidders to invert bid functions to infer an ascendingly ordered vector \((x_1, \ldots, x_n)\) of the cost estimate realizations. Let \(\Psi = E[C_0 | X_1 = x_1, \ldots, X_n = x_n]\), which is the single post-auction estimate of \(C_0\) that all bidders can calculate. Next, let \(d_i = \lambda \Psi + (1 - \lambda)X_i\) be the post-auction estimate of bidder \(i\)'s cost of fulfilling the contract. Note that every bidder has the information to calculate \(d_i\) for any bidder.

Now if \(d_i < B_1\), bidder 1 expects a positive profit, even if \(B_1 < \Psi\), and so does not withdraw. When \(d_i > B_1\), bidder 1 expects to lose money by fulfilling the contract. However, if \(d_i < B_2\), he expects a smaller loss than the compensation penalty. Hence, \(d_i > B_2\) is bidder 1's withdrawal condition, and analogously \(d_i > B_{i+1}\) is bidder \(i\)'s withdrawal condition should he be offered the contract. As \(d_i > d_{i-1}\), bidder \(i\) expects a loss due to contract fulfillment whenever bidder \(i - 1\) withdraws and the buyer turns to him. Unlike the pure common-value model, however, it is possible that some bidder chooses not to withdraw even when the next lowest bid is below \(\Psi\). One example would arise if \(B_2 < d_1 < d_2 < B_3 < \Psi\), in which case any of the three lowest bidders would expect a loss if called upon, but only the lowest bidder chooses to withdraw.

Note that in the common value model considered above and the affiliated value model just discussed the bid-taker benefits from allowing bid withdrawals with compensation penalties. This follows trivially from the facts that bidding is more aggressive and that the bid-taker is unharmed by withdrawals.

With affiliated values, compensation penalties create an interesting incentive for side payments from one bidder to induce another not to withdraw. Suppose that the lowest bidder (1) would choose to withdraw, the buyer would definitely turn to the second-lowest bidder (2) upon this withdrawal, and bidder 2 would then choose to fulfill the contract. Bidder 2 would then prefer to offer bidder 1 a side payment \(s < d_2 - B_2\) in return for bidder 1 not withdrawing. Bidder 1 would prefer fulfilling the contract and receiving the side payment over withdrawing if \(s > d_1 - B_2\). Because bidder 1 has a cost advantage in fulfilling the contract, there is a range of side payments \(s\) which are mutually beneficial. Note that, at the stage when bargaining over such a side payment would occur, the two bidders’ private information has been revealed by the bids, so the bargaining would be under circumstances of complete information. The analysis in this paper assumes that either such side payments are not possible or that bidders do not alter their bids to affect the possible side payment negotiations. In the event that \(\alpha < 1\), how any side payment scheme would work, if at all, would depend upon the precise timing of the negotiations and the bidders learning the buyer’s decision as to whether to award the contract to bidder 2 in the event bidder 1 withdraws.

Finally, a corresponding analysis would conclude that a seller can benefit from offering bid withdrawals under compensation penalties in a sealed bid auctions.

4. A PARTICULAR MODEL

To show the strategic richness possible even in a simple model with bid withdrawals, we now present a multi-cumulative common-value model. In it, each of \(n\) bidders receives a cost estimate drawn independently from the same commonly known estimating distribution. There is a penalty for withdrawing a bid that we assume to be a fraction \(p\) of the withdrawn bid (e.g., the forfeiture of a 10% bid bond) and a probability \(\alpha\) that the bid-taker will award the contract to the second lowest bidder if the lowest bid is withdrawn. Let \(c\) denote the unknown common cost. For \(i = 1, 2, \ldots, n\) let \(b_i(c_i)\) be the bid selected by bidder \(i\) when his estimate is \(c_i\), let \(w_i(b_1, b_2, \ldots, b_n)\) be chosen by bidder \(i\) so that he will withdraw his bid if the best competitive bid is more than \(b_iw_i\), let \(j\) index the best competitive bid, let \(k\) index the second-best competitive bid, let \(F_j(b_i)\) be the cumulative distribution of bidder \(i\)'s bid (obtainable by combining the bid strategy with the estimating distribution), and let \(E_i\) be bidder \(i\)'s expected profit. With these definitions and standard notation for probabilities and for Lebesgue–Stieltjes integration, we have
\[ E_i = \int_0^\infty (b_i - c) \Pr(b_i \leq b_j \leq w_i b_i) dF_i(b_i) \]
\[ - p \int_0^\infty b_j \Pr(b_j \geq w_i b_i) dF_i(b_i) \]
\[ + \alpha \int_0^\infty (b_i - c) \Pr(w_j b_j \leq b_i \leq b_k) dF_i(b_i). \]

Next, we specialize this model to a variant of one first considered by Rothkopf (1969) in which the estimating distribution is the Weibull distribution, the strategies are multiples (rather than general functions) of the estimates,^9 and hence the bids also have a Weibull distribution. This bid distribution can be parameterized as

\[ F(b_i) = 1 - e^{-a_i b_i^m}, \]

where \( m \) is the parameter that controls the standard deviation to mean ratio of the distribution,^10 \( S_i \) is the strategy factor by which bidder \( i \) multiplies his unbiased estimate to obtain his bid, and \( a_i \) is

\[ a_i = [\Gamma(1 + 1/m) / S_i c]^m. \]

Specializing now to the two-bidder case (i.e., \( n = 2 \)), allows us to treat \( w_i \) as a constant (chosen by bidder \( i \)) rather than a function of \( n - 1 \) variables without further loss of generality.

Within this model,
\[ \Pr(b_i \leq b_j \leq w_i b_i) = \Pr(b_j \leq w_i b_i) - \Pr(b_j \leq b_i) = e^{-a_i b_i^m} - e^{-a_i w_i b_i^m}, \]

\[ \Pr(b_j \geq w_i b_i) = e^{-a_i w_i b_i^m}, \]

\[ \Pr(w_i b_j \leq b_i \leq b_k) = \Pr(w_i b_j \leq b_i) < \infty \]
\[ = 1 - e^{-a_i w_i b_i^m}, \]

and hence,
\[ E_i = a_i (a_i + a_j)^{-1-1/m} \Gamma - (1+p) a_i (a_i + a_j w_i^m)^{-1-1/m} \Gamma \]
\[ - c a_i (a_i + a_j)^{-1} + c a_i (a_i + a_j w_i^m)^{-1} \Gamma \]
\[ + \alpha [a_i^{-1-1/m} \Gamma - a_i (a_i + a_j w_j^{-m})^{-1-1/m} \Gamma \]
\[ - c a_i (a_i + a_j w_j^{-m})^{-1} - c a_i (a_i + a_j w_j^{-m})^{-1}], \]

where \( \Gamma \) denotes the gamma function with argument \((1 + 1/m)\).

Setting \( \partial E_i / \partial w_j = 0 \), using the symmetry conditions \( w_i = w, S_i = S \) and \( a_i = (\Gamma / S c)^m, i = 1, 2, \) and solving yields the symmetric equilibrium bid withdrawal policy
\[ w = \left[ S^m (1+p) / (1 + 1/m) \right]^{-1/m}. \]

(4)

Setting \( \partial E_i / \partial S_i = 0 \) and using the symmetry conditions yields:
\[ S = \left[ 2^{-2} - (1 + \alpha) / (2 + w^{-m} + w^m) \right] \]
\[ \cdot \left[ (1 - 1/m) 2^{-2-1/m} (1 + p) (w^{-m} - 1/m) \right] \]
\[ \cdot (1 + w^{-m})^{2-1/m} - \alpha (w^{-m} - 1/m) \]
\[ \cdot (1 + w^{-m})^{2-1/m} - \alpha / m \].

Table I

<table>
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In general, (4) and (5) must be solved together numerically. Notice, however, that when \( p, \) the penalty factor for bid withdrawals, is large, \( w \) is also large, and (5) can be simplified to obtain the two-bidder version of the solution in Rothkopf (1969) for the case in which bid withdrawals are not allowed: \( S = m^2 1/m ((1 + 1/m) - 1) \). Also, when \( \alpha = 0, i.e., \) when a withdrawn bid will never lead to a contract with another bidder, substituting (4) into (5) yields a polynomial equation for \( S \):
\[ 2^{-1/m} (1 + 1/m) S^{m+1} - m S^m + 4(1+p)^{-m} (1 + 1/m) S^{-1/m} \]
\[ = 0. \]

(6)

Table I gives values for \( S \) determined for selected values of \( m \) and \( p \) using (6). Table II gives the corresponding value of \( w \) obtained by using the result from (6) in (4).

Table II

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In the symmetric two bidder equilibrium, this is \( 1 / (1 + w^m) \) or, equivalently, \( [S (1 + p) (1 + 1/m)]^{-m} \). In the
Table III
Normalized Equilibrium Expected Profit for Each Bidder, E/c
\[ \alpha = 0 \]

\[ \begin{array}{cccccc}
\text{m} & 0 & 0.01 & 0.1 & 0.2 & 0.5 & 1.0 \\
p & & & & & & \\
\hline
1.5 & 0.898 & 0.900 & 0.914 & 0.925 & 0.948 & 0.967 & 1.000 \\
2 & 0.433 & 0.435 & 0.447 & 0.457 & 0.473 & 0.486 & 0.500 \\
4 & 0.138 & 0.139 & 0.149 & 0.155 & 0.162 & 0.165 & 0.167 \\
8 & 0.058 & 0.059 & 0.066 & 0.069 & 0.071 & 0.071 & 0.071 \\
16 & 0.027 & 0.028 & 0.032 & 0.033 & 0.033 & 0.033 & 0.033 \\
32 & 0.013 & 0.014 & 0.016 & 0.016 & 0.016 & 0.016 & 0.016 \\
64 & 0.006 & 0.007 & 0.008 & 0.008 & 0.008 & 0.008 & 0.008 \\
\end{array} \]

two-bidder case, when the winning bid is withdrawn the remaining bidder’s evaluation of the cost of the job will always decrease, so he prefers to accept an award at the price he offered if it is available. Hence, the probability that the buyer will not have the work done by any bidder, \( \Omega \), is \( 2(1 - \alpha)[S(1 + p)(1 + 1/m)]^{-m} \). Table IV gives the value of this quantity for the \( \alpha = 0 \) cases considered in Tables I–III. It shows the probability of a withdrawal by the winning bidder in the absence of a withdrawal penalty ranging from about 10–20% with the larger probabilities associated with more accurate estimating. As the withdrawal penalty is increased, these probabilities drop—precipitously in the case of more accurate estimating.

Tables V through VIII give information similar to that in Tables I–IV for several nonzero values of \( \alpha \). The possibility of an award to the higher bidder (when his bid is especially high relative to the low bid) has a dramatic effect. It leads to much less aggressive bidding and much higher expected profits as long as the penalty for bid withdrawal is not prohibitive. For example, when \( m = 8, p = 0 \), and \( \alpha \) goes from 0 to 1, the equilibrium markup goes from 19.4% to 34.8% and the expected profit of each bidder increases from 5.8% of the cost of the job to 14% of it.

The richness remaining in a two-bidder multiplicative model shows up in the way direction relative statics hinge on parameters. For example, consider the way

Table IV
Equilibrium Probability that Contract is Not Performed, \( \Omega \)
\[ \alpha = 0 \]

\[ \begin{array}{cccccc}
\text{m} & 0 & 0.01 & 0.1 & 0.2 & 0.5 & 1.0 \\
p & & & & & & \\
\hline
1.5 & 0.1016 & 0.0998 & 0.0863 & 0.0746 & 0.0519 & 0.0329 \\
2 & 0.1340 & 0.1307 & 0.1064 & 0.0869 & 0.0531 & 0.0289 \\
4 & 0.1725 & 0.1631 & 0.1051 & 0.0700 & 0.0267 & 0.0082 \\
8 & 0.1887 & 0.1675 & 0.0709 & 0.0330 & 0.0053 & 0.0005 \\
16 & 0.1957 & 0.1543 & 0.0312 & 0.0074 & 0.0002 & 0.0000 \\
32 & 0.1997 & 0.1256 & 0.0065 & 0.0004 & 0.0000 & 0.0000 \\
64 & 0.2014 & 0.0846 & 0.0003 & 0.0000 & 0.0000 & 0.0000 \\
\end{array} \]

in which equilibrium bidder aggressiveness responds to increases in \( p \), the penalty for withdrawing. For low values of \( \alpha \), as in Table I or the upper part of Table V, higher penalties yield less aggressive bidding. This is because the sole effect of larger penalties is to make the insurance aspect of bid withdrawals more costly to use. If \( \alpha \) is as large as 0.1, however, the response to larger penalties switches to more aggressive bidding. The reason is that with larger \( \alpha \) and small \( p \) the focus is not so much on the opportunity to withdraw as on the extremely profitable opportunity presented when the rival bidder withdraws and the higher bid is honored. If \( p \) is large, the rival rarely presents this opportunity, and the way to profit shifts back to winning the auction.

Multiplicative auction purchase models such as this one have mirror image auction sale models (see Rothkopf 1969, 1991). In the sale models, the Weibull random variable is replaced by its reciprocal (which has a Gumbel distribution), and all of the analytical results have analogies. There is nothing about this model to suggest that this will not be so, but we have not analyzed formally the mirror image model.

Next, we consider whether, in the context of this purchase model, a rational bid-taker would ever choose to invoke explicitly a bid withdrawal option. The bid-taker
Table VI
Equilibrium Bid Withdrawal Factor, \( w \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.01 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>7.239</td>
<td>8.075</td>
<td>11.363</td>
<td>15.413</td>
</tr>
<tr>
<td>2</td>
<td>3.796</td>
<td>4.274</td>
<td>6.093</td>
<td>8.288</td>
</tr>
<tr>
<td>4</td>
<td>1.816</td>
<td>2.068</td>
<td>2.924</td>
<td>3.948</td>
</tr>
<tr>
<td>8</td>
<td>1.331</td>
<td>1.512</td>
<td>2.100</td>
<td>2.804</td>
</tr>
<tr>
<td>16</td>
<td>1.150</td>
<td>1.296</td>
<td>1.775</td>
<td>2.367</td>
</tr>
<tr>
<td>64</td>
<td>1.035</td>
<td>1.147</td>
<td>1.564</td>
<td>2.086</td>
</tr>
</tbody>
</table>

| \( \alpha = 0.10 \) |
| 1.5  | 8.822 | 9.579 | 12.644 | 16.520 |
| 2    | 4.300 | 5.143 | 6.408 | 8.520 |
| 4    | 1.910 | 2.130 | 2.955 | 3.957 |
| 8    | 1.362 | 1.524 | 2.101 | 2.804 |
| 16   | 1.163 | 1.298 | 1.775 | 2.367 |
| 64   | 1.038 | 1.147 | 1.564 | 2.086 |

| \( \alpha = 0.5 \) |
| 1.5  | 14.093 | 14.735 | 17.386 | 20.844 |
| 2    | 5.843 | 6.174 | 7.574 | 9.445 |
| 4    | 2.174 | 2.334 | 3.041 | 3.994 |
| 8    | 1.446 | 1.576 | 2.106 | 2.805 |
| 16   | 1.198 | 1.306 | 1.775 | 2.367 |
| 64   | 1.045 | 1.147 | 1.564 | 2.086 |

| \( \alpha = 1.0 \) |
| 1.5  | 19.149 | 19.746 | 22.210 | 25.432 |
| 2    | 7.186 | 7.479 | 8.728 | 10.433 |
| 4    | 2.380 | 2.512 | 3.136 | 4.039 |
| 8    | 1.509 | 1.608 | 2.112 | 2.805 |
| 16   | 1.223 | 1.314 | 1.775 | 2.367 |
| 64   | 1.051 | 1.147 | 1.564 | 2.086 |

Table VII
Normalized Equilibrium Expected Profit for Each Bidder, \( E/c \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.01 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.937</td>
<td>0.947</td>
<td>0.968</td>
<td>0.980</td>
</tr>
<tr>
<td>2</td>
<td>0.449</td>
<td>0.459</td>
<td>0.480</td>
<td>0.489</td>
</tr>
<tr>
<td>4</td>
<td>0.142</td>
<td>0.152</td>
<td>0.163</td>
<td>0.166</td>
</tr>
<tr>
<td>8</td>
<td>0.060</td>
<td>0.067</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>16</td>
<td>0.028</td>
<td>0.032</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>64</td>
<td>0.007</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
</tr>
</tbody>
</table>

| \( \alpha = 0.10 \) |
| 1.5  | 1.253 | 1.221 | 1.142 | 1.093 |
| 2    | 0.575 | 0.563 | 0.535 | 0.520 |
| 4    | 0.175 | 0.173 | 0.169 | 0.167 |
| 8    | 0.072 | 0.072 | 0.072 | 0.071 |
| 16   | 0.033 | 0.033 | 0.033 | 0.033 |
| 64   | 0.008 | 0.008 | 0.008 | 0.008 |

| \( \alpha = 0.5 \) |
| 1.5  | 2.326 | 2.177 | 1.788 | 1.537 |
| 2    | 0.976 | 0.908 | 0.741 | 0.644 |
| 4    | 0.273 | 0.245 | 0.192 | 0.175 |
| 8    | 0.110 | 0.091 | 0.073 | 0.072 |
| 16   | 0.050 | 0.038 | 0.033 | 0.033 |
| 64   | 0.012 | 0.008 | 0.008 | 0.008 |

| \( \alpha = 1.0 \) |
| 1.5  | 3.367 | 3.115 | 2.451 | 2.010 |
| 2    | 1.335 | 1.225 | 0.947 | 0.777 |
| 4    | 0.355 | 0.309 | 0.218 | 0.184 |
| 8    | 0.140 | 0.110 | 0.075 | 0.072 |
| 16   | 0.063 | 0.042 | 0.033 | 0.033 |
| 64   | 0.015 | 0.008 | 0.008 | 0.008 |

must pay the cost of the job if it is done and the expected profit of each bidder. In addition, he will bear a cost that is presumably at least as great as the cost of the job if the job is not done by one of the bidders. Offsetting these costs is whatever fraction of the withdrawal penalties paid by the bidders that goes to the bid-taker (as opposed, for example, to being the cost of reputation loss). Thus, in the symmetric two-bidder equilibrium we have examined, the bid-taker's expected net cost, \( N \), a function of \( \alpha \) and \( p \), is given by

\[
N = c + 2E + \Omega C^*c - 2\zeta \pi ,
\]

where \( C^* \) is the multiple of contract cost necessary to get the work done without use of the services of the bidders, \( \pi \) is each bidder's expected penalty cost, and \( \zeta \) is the fraction of these that go to the bid-taker. Clearly, if \( C^* \) is sufficiently large, the bid-taker's net cost will be minimized by setting \( \alpha = 1 \) or by making \( p \) so large that no bid withdrawals will occur.

However, despite the fact that important aspects of the cost work against bid withdrawals being in the bid-taker's interest (i.e., only two bidders and no bidder risk aversion) there are values of the model's parameters for which the bid-taker prefers to allow bid withdrawals even when \( C^* \) is large. For example, when \( m = 64 \), the bid-taker can be better off with policies that allow some withdrawals. If the bid-taker sets \( \alpha = 1 \), he prefers values of \( p \) in the following decreasing order: 0.2, 0.5, 1, \( \infty \), 0.1, 0.01, and 0, where \( p = \infty \) represents not allowing withdrawals. For these parameters, net cost increases only slightly as \( p \) is increased from 0.2 (the increase does not occur in the first four significant digits). However, lowering \( p \) below 0.2 is costly to a comparatively dramatic degree. What is happening with these parameters? Lowering \( p \) from 1 to 0.2 increases the probability of a withdrawal by a factor of about 1.5, up to about 1%. Further reductions in \( p \) do not markedly raise this probability. (It still has not risen to 1.2% for \( p = 0 \) ) Allowing withdrawals in the 1% of cases where a bidder's rival's cost estimate turns out to be at least 25% higher than the lower bidder's estimate is valuable to the bidders. They respond by bidding about 0.01% more aggressively. They would continue to bid increasingly aggressively if \( p \) were lowered below 0.2. However, the bid-taker would be paying the higher bid somewhat more frequently and with insufficient compensating penalty receipts.
Table VIII
Equilibrium Probability That Contract is Not Performed, Ω

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.01$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.0967</td>
<td>0.0827</td>
<td>0.0504</td>
<td>0.0322</td>
</tr>
<tr>
<td>2</td>
<td>0.1285</td>
<td>0.1028</td>
<td>0.0519</td>
<td>0.0284</td>
</tr>
<tr>
<td>4</td>
<td>0.1666</td>
<td>0.1026</td>
<td>0.0264</td>
<td>0.0081</td>
</tr>
<tr>
<td>8</td>
<td>0.1827</td>
<td>0.0997</td>
<td>0.0052</td>
<td>0.0005</td>
</tr>
<tr>
<td>16</td>
<td>0.1901</td>
<td>0.0308</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>64</td>
<td>0.1955</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

| $\alpha = 0.10$ | | | | |
| 1.5 | 0.0661 | 0.0587 | 0.0392 | 0.0264 |
| 2   | 0.0924 | 0.0773 | 0.0656 | 0.0428 |
| 4   | 0.1258 | 0.0834 | 0.0233 | 0.0073 |
| 8   | 0.1404 | 0.0597 | 0.0047 | 0.0005 |
| 16  | 0.1473 | 0.0273 | 0.0002 | 0.0000 |
| 64  | 0.1522 | 0.0003 | 0.0000 | 0.0000 |

| $\alpha = 0.5$ | | | | |
| 1.5 | 0.0186 | 0.0174 | 0.0136 | 0.0104 |
| 2   | 0.0285 | 0.0256 | 0.0171 | 0.0111 |
| 4   | 0.0428 | 0.0326 | 0.0116 | 0.0039 |
| 8   | 0.0497 | 0.0267 | 0.0026 | 0.0003 |
| 16  | 0.0529 | 0.0138 | 0.0001 | 0.0000 |
| 64  | 0.0553 | 0.0002 | 0.0000 | 0.0000 |

5. DISCUSSION

Models are necessarily less complex than the situations they seek to describe and analyze. When a model achieves some measure of success in a limited domain, a natural step is to seek to broaden the domain by relaxing some of the model’s assumptions. In an effort to broaden the relevance of auction theory to markets where buyers engage in roughly auction-like negotiation processes, a useful step is to weaken auction models’ assumption of irrevocable commitments. This paper attempts that step on one side of the market by introducing explicitly the possibility that the most competitive bidder might choose on occasion to back out of the commitment implied by his bid. That is, in our models, such commitments are tentative and enforced only to a limited extent.

While beginning in this way to extend auction theory to settings encompassing more negotiation-like flexibility, our model stays strictly game-theoretic. No bidder withdraws an otherwise winning bid because it was mistakenly or naively submitted. In our analysis, each bidder submits a bid that leads to positive expected profit given that it wins. Thus, all bidders make sufficient allowance, on the average, for the winner’s curse. Nonetheless, upon learning not only that he was the low bidder, but also the particular bids made by rivals, there are situations in which the rationally submitted bid is now rationally withdrawn, even when bid withdrawal is costly. Indeed, bid withdrawal occurs precisely when the expected loss upon learning all bids is greater than the withdrawal penalty. It is entirely possible that a bid-taker would be able to collect a risk premium from risk-averse bidders by offering this kind of winner’s curse insurance. While we have ruled out this motivation by assuming risk-neutral bidders, nevertheless, bidders bid more aggressively in response to the presence of a withdrawal option, unless the opportunity to obtain a contract as the next best bidder after a bid withdrawal is sufficiently likely and lucrative. Section 2 provides the details.

While the lowest bid will be lower with withdrawable bids, this is not sufficient to conclude that the bid-taker expects to gain by allowing withdrawals. When the lowest bidder withdraws, the bid-taker may have to pay more for the contract, unless the withdrawal penalty received compensates for the added cost. We have shown that the particular form of withdrawable bid auction in which bidders are committed to compensate the bid-taker exactly for the added cost of their withdrawal is preferable for the bid-taker to standard sealed bidding without withdrawals. That this is so is an interesting reflection on the wisdom of the common law doctrine on mitigation of damages for breach of contract.

While compensation penalty rules are attractive in theory, they may not be practical. In practice, commitments by bidders frequently take the form of a fixed percentage of the bid, as in a bid bond. Bidders may be reluctant to rely on a bid withdrawal option when the withdrawal penalty is unknown at the time of bid submission. The fixed fraction-of-bid withdrawal penalty is a cornerstone of the two-bidder Weibull distribution, multiplicative model we solve in Section 4. As those results indicate, the model offers a richness of comparative statics that suggests steps toward the analysis of more complex competitive processes.

Riley (1988) models an auction in which the winner’s payment is a weighted sum of the two best bids and shows that such auctions lead to more aggressive bidding and better expected results for the bid-taker than does standard sealed bidding. The reason for this is that it takes account of the private information of the second best bidder in setting the price. This is also a characteristic of the withdrawable bid auctions we study.

We close with a caveat. The results presented here have been developed in an exploration of the consequences of the standard game-theoretic assumptions of common bidding models rather than in response to an analysis of observed auction behavior. In particular, note that the standard game theory assumptions we use imply that upon learning a bidder’s bid other bidders know his signal precisely—i.e., everything he knows about the value of what is being sold. It is worth considering what the effect would be on our conclusions were his private information revealed only imprecisely. As the information a bidder can obtain by inverting rivals’ bid functions becomes less useful, we would expect that the opportunity to withdraw a bid should be less frequently used and less significant than the models presented here.
predict. However, a number of our general and qualitative results should still hold. The condition given for the rational withdrawal of a bid will be slightly generalized by replacing the expectation in the right-hand side of (1) by \( E[c|B_j], j = 1, 2, \ldots, n \). The theorem giving a sufficient condition for more aggressive bidding is essentially unchanged, and its three corollaries still hold. The analysis of bidding with compensation penalties will remain qualitatively the same, but there will no longer be a common agreed-upon post-auction cost estimate.

To the extent that conclusions from our models prove to be unrealistic, some conclusions from standard models may also be called into question. To cite but one example, the concerns just discussed, over inferring private information reliably by inverting bid functions, also suggest caution in evaluating the model of English auctions proposed in Milgrom and Weber (1982), which crucially depends upon such information. In another paper, Harstad and Rothkopf (1991) show that the expected revenue calculations for English auctions in Milgrom and Weber are overstated if this information is not available.

**APPENDIX**

**A.1. Definitions**

Recall that symmetry allows a focus on bidder 1, who will be presumed to have observed \( X_1 = x \in \mathcal{X} = \text{supp}[X_1] \), \( Y := \min\{X_j, j \neq 1\} \), and let \( Z = \{X_2, \ldots, X_n\} \setminus \{Y\} \). Since conditioning variables are listed explicitly, we abuse notation slightly by denoting any conditional distribution of \( V \) given \( V' = v' \) as \( F_v(v|v') \), for any mutually exclusive vectors \( V, V' \), and denote the associated density by \( f_v \). Throughout the Appendix, \( p \geq 0 \) is fixed arbitrarily, and \( b(\cdot) \) denotes the symmetric equilibrium of the first-price auction. Define:

\[
L^+(x, y, \delta) := \left\{ z \in \mathcal{X}^{n-2} | \delta \geq \int_{\mathcal{X}} cdF_C(c|x, y, z) \right\},
\]

\[
L^0(x, y, \delta) := \left\{ z \in \mathcal{X}^{n-2} | \delta = \int_{\mathcal{X}} cdF_C(c|x, y, z) \right\},
\]

\[
L^-(x, y, \delta) := \left\{ z \in \mathcal{X}^{n-2} | \delta \leq \int_{\mathcal{X}} cdF_C(c|x, y, z) \right\}.
\]

In words, given \( X_1 = x \) and \( Y = y \), \( L^+ \) [respectively,\( L^0, L^- \)] is the set of realizations of signals of the remaining \( n - 2 \) bidders for which the expected cost of contract fulfillment is at most \( \delta \) [respectively, exactly \( \delta \), at least \( \delta \)]. Let the bid which bidder 1 would submit in a first-price auction, \( b(x) \), be abbreviated \( B \).

Next we define constituent parts of the impacts used in condition (2) in the text. Consider a symmetric equilibrium in which bidder 1 submits the bid \( B \). The equilibrium probability that he will withdraw that bid is

\[
\rho_w(x, p) := \int_{x}^{\infty} f_y(y|x) \int_{L^-(x,y,B+p)} f_Z(z|x, y) \, dz \, dy.
\]

Expected marginal avoided cost due to withdrawing is

\[
A(x, p) := \int_{\mathcal{X}} \int_{L^-(x,x,B+p)} cf_{cz}(c, z|x, x) \, dz \, dc.
\]

To clarify, consider the costs that bidder 1 would have been required to bear could he not withdraw, but which he avoids by making an optimal decision as to when to withdraw. \( A(x, p) \) is the change in expectation of such costs as 1 bids incrementally more aggressively, and thus wins marginally more often. The first impact the text discusses can then be defined:

\[
I_a(x, B, p) := A(x, p) f_y(x|x) + \rho_w(x, p).
\]

To obtain condition (2), we will initially suppose that \( (b, \ldots, b) \), the first-price equilibrium, is an equilibrium in the withdrawable bid context, and derive the contradiction that responding more aggressively raises bidder 1's expected profit. This initial supposition is used to simplify the expression of the remaining three definitions. Given \( X_1 = x \) and \( p \geq 0 \), \( \theta(x, p) \) is the event that 1 submits the second-lowest bid and the lowest bidder is indifferent between performing the contract and paying \( p \) to withdraw. The probability density of this event is

\[
\rho_T(x, p) := \int_{x}^{\infty} \int_{L^-(x,y,b(y)+p)} f_y(y, z|x) \, dz \, dy,
\]

and the expected lowest bid in this event is

\[
T(x, p) := \int_{x}^{\infty} \int_{L^-(x,y,b(y)+p)} b(y) f_y(y, z|x) \, dz \, dy / \rho_T(x, p).
\]

The second impact discussed in the text is then defined as

\[
I_b(x, B, p) := \alpha \rho_T(x, p)[b(x) - p - T(x, p)].
\]

**A.2. Principal Result**

**Theorem.** A sufficient condition for a symmetric equilibrium of a withdrawable bid auction to exhibit more aggressive bidding than a first-price auction is

\[
A(x, p) f_y(x|x) + \rho_w(x, p) - \alpha \rho_T(x, p)
\]

\[
\cdot [b(x) - p - T(x, p)] > 0, \quad \text{for all } x \in \mathcal{X}. \quad (2')
\]

**Proof.** Suppose that all bidders use the first-price equilibrium bid function \( b(\cdot) \). If bidder 1 unilaterally reduces his bid from \( B \) to \( B - dB \), his expected gain is (ignoring terms of the order of magnitude of \( dB^2 \)):
\[ \gamma = \left[ B - \int_{\mathcal{Z}C} \int_{L_{x,y}^+(x,y,B+y+p)} cf_{CYZ}(c, z|x, x) \, dz \, dc \right] \cdot f_Y(x|x) \\
- \int_x^\infty f_Y(y|x) \int_{L_{x,y}^+(x,y,B+y+p)} f_Z(z|x, y) \, dz \, dy \\
- \alpha (B - \int_{\mathcal{Z}C} \int_{L_{x,y}^+(x,y,B+y+p)} f_Y(y|x) \, dz \, dy + \int_x^\infty f_Y(y|x) \int_{\mathcal{Z}C} f_Z(z|x, y) \, dz \, dy) \cdot (1 - 1_{L_{x,y}^+(x,y,B+y+p)}) \, dy =: \xi \]

(A.1)

In (A.1), the first term reflects the incremental increase in the frequency of winning, for those cases in which 1 does not want to withdraw. The second term reflects the incremental reduction in compensation in those cases where \( B \) was a sufficiently low bid to win, and 1 does not want to withdraw. The last term reflects the fact that, if bidder 1 bids slightly more aggressively, there is some possibility that 1 is the second-lowest bidder, and the lowest bidder would have slightly preferred to withdraw if he had inferred bidder 1’s information to be \( b^{-1}(B) \), but does not withdraw when inferring 1’s information to be \( b^{-1}(B) - dB \). For convenience, refer to the first two of the three terms in (A.1) as \( \gamma_1 \), and the third term as \( \gamma_2 \), so that \( \gamma = \gamma_1 + \gamma_2 \).

The theorem asserts that (2’) implies \( \gamma > 0 \). Since the only event in which an incremental reduction in 1's bid could prevent a withdrawal is when the lowest bidder was essentially indifferent over withdrawing, \( \gamma_2 \) can be simplified by concluding that, for this event, in expectation, \( C = b(y) + p \):

\[ \gamma_2 = -\alpha \left( \int_{-\infty}^x \int_{L_{x,y}^+(x,y,b+y+p)} \right) [B - p - b(y)] f_Y(y, z|x) dz \, dy \]

(A.2)

\[ = -\alpha \rho_T(x, p) [b(x) - p - T(x, p)]. \]

Since \( b \) is assumed to be the first-price equilibrium, in a first-price auction, bidder 1 would find that the expected gain from decreasing his bid by \( dB \) is zero:

\[ 0 = \left[ B - \int_{\mathcal{Z}C} cf_C(c|x, x) dc \right] f_Y(x|x) \\
- \int_x^\infty f_Y(y|x) dy =: \xi. \]

(A.3)

Subtracting \( \xi \) from \( \gamma_1 \) yields

\[ \gamma_1 = \int_{\mathcal{Z}C} \int_{\mathcal{Z}Z} cf_{CYZ}(c, z|x, x) \, dz \, dy + \int_x^\infty f_Y(y|x) \int_{\mathcal{Z}Z} f_Z(z|x, y) \, dz \, dy \]

\[ \cdot (1 - 1_{L_{x,y}^+(x,y,B+y+p)}) \, dz \, dy =: \zeta \]

= \alpha (x, p) f_Y(x|x) + \rho_T(x, p). \]

(A.4)

Combining (A.2) and (A.4), bidder 1 gains by unilaterally bidding more aggressively in response to \( (b, \ldots, b) \) if and only if (2’) holds. Hence, \( (b, \ldots, b) \) cannot be an equilibrium. If an increasing bid function \( b \) with \( b(x) > b(x) \) on a non-null subset of \( \text{supp}[X_1] \) were supposed to be an equilibrium, a contradiction would be reached by a similar argument: (A.2) repeated, \( \xi > 0 \), so \( \zeta = \gamma_1 - \gamma_1 > \xi \), and (2’) implies \( \xi > 0 \).

### A.3. Sketch of Implications

**Implication i.** For given \( p \geq 0 \), underlying distribution \( \mathcal{D} \) of the model’s random variables, and \( n, \mathcal{F} \in (0, 1] \), so that \( \alpha < \mathcal{F} \Rightarrow (2’) \).

**Trivial:** The first two terms in (2’) are positive, and \( \alpha \) appears only as a coefficient of the third term.

**Implication ii.** For given \( p \geq 0 \), \( \mathcal{D} \), \( \alpha \in [0, 1] \), \( \mathcal{F} < \infty \), so that \( n > \mathcal{F} \Rightarrow (2’) \).

It is tedious but straightforward to show that the ratio of the third term in (2’) to the second term in (2’) converges to 0 as \( n \) approaches \( \infty \). Intuitively, the second term contains a probability that bidder 1 submits the lowest bid, and thus is of the rough order of magnitude \( n^{-1} \), while the third term requires a particular relationship between the lowest signal and bidder 1’s signal, which must be second-lowest, and is thus of the rough order of magnitude \( n^{-2} \).

**Implication iii.** Given \( p \geq 0 \), \( n, \alpha \in [0, 1] \), for many underlying distributions \( \mathcal{D} \) of the model’s random variables, sufficient variability of \( F_{X_1}(x|c) \Rightarrow (2’) \).

Consider a sequence \( \Pi = \{ \mathcal{D}^k \}_{k=1,2,\ldots} \) of members of some parametric family \( \mathcal{P} \) of distributions. For simplicity, assume that \( \mathcal{F} \) is unbounded above, and \( X_1 \) is an unbiased estimator of \( C \). Let \( \Pi \) be ordered so that \( F_{X_1}^{k+1}(x|c) \) is a mean-preserving spread of \( F_{X_1}^k(x|c) \). The limitation on distributions used for the comparative static is that this ordering of \( \Pi \) also implies that \( F_{C_1}^k(c|x) \) is a mean-preserving spread of \( F_{C_1}^{k+1}(c|x) \). This is not a strong requirement, presumably satisfied by distributional families with unimodal conditionals. Indeed, unboundedness and unbiasedness can be finessed. With mean-preserving spreads going both ways as the model is switched from
\( \mathcal{X}^k \) to \( \mathcal{X}^{k+1} \), eventually the first term in (2') is substantially increased without measurably increasing (in absolute value) the third term.

NOTES

1. See Rothkopf (1991) for an example.
2. See Ashtenfelter and Genesove (1992) for a discussion of the incentive for such defaults in real estate auctions.
3. Bidding theory began with independent private values models (see Friedman 1956 and Vickrey 1961) in which the bids of competitors were independent random variables from the value to a bidder of the subject of the auction and thus contained no information about this value. Later, common value models entered the literature (see Wilson 1967, 1977, Rothkopf 1969, and Capen, Clapp and Campbell 1971). In these models, there is an unknown common value that is estimated independently by the bidders. Hence, a rival’s bid formulated using a known strategy reveals information to a bidder about the value of what he is bidding on. Furthermore, the winner in a common value auction will normally be the maker of the most optimistic value estimate. Capen, Clapp and Campbell are responsible for pointing out the importance of this selection bias and moving the colorful phrase to describe it, “winner’s curse,” from the oil patch to the economics literature. In 1982, Milgrom and Weber developed a general symmetric model for “affiliated values.” Affiliated values span the range from independent private values to common values.
4. The assumption of a unique symmetric equilibrium in which the equilibrium bid function is strictly increasing in the bidder’s information is ubiquitous and unchallenged in the auction theory literature. We are not aware of any published proofs justifying this assumption for sealed-bid auctions. Maskin and Riley’s (1986) working paper has a complex proof for sealed-bid auctions, which assumes the underlying distribution of signals is nonatomic and affiliated. The methodology of the proof could be followed, clearly with added complexity, for auctions with bid withdrawal.
5. Unless this probability \( \alpha \) is 0 or 1, it is potentially a serious restriction that it is independent of the amounts of the bids. For some purposes, it could be more appropriate to treat \( \alpha \) as a decision variable with an exogenous upper bound, reflecting such factors as the passage of enough time in approaching the next bidder that he has gone on to make other commitments inconsistent with fulfilling this contract at his bid price.
6. In particular, the condition presented here holds whether or not a second lowest bid can also be withdrawn, provided that such a withdrawal does not result in an award to any other bidder. Note also that Section 3 considers a model with a different penalty structure in which any bid except the highest may be withdrawn.
7. Here, we are ignoring the complications caused by ties. In particular, if there is a bid exactly at the common post-auction estimate, the bidder immediately below is indifferent about withdrawing.
8. We call a distribution pathological if the first-price (i.e., withdrawal-not-allowed) symmetric equilibrium bid function remains above the supremum of the support of the cost distribution given the signal. We are not aware of any such examples with cost uncertainty. The conclusion of more aggressive bidding follows from the fact that no bidder gains from a rival’s withdrawal.
9. When the prior distribution of \( c \) is diffuse, the use of multiplicative strategies is not restrictive. For many families of priors, the effect of the use of multiplicative strategies disappears in the limit as the prior becomes diffuse. For an example, see Rothkopf (1980b). See Rothkopf (1980a), Engelbrecht-Wiggans (1979), and Engelbrecht-Wiggans and Weber (1979) for further discussion of the restrictiveness of the use of multiplicative strategies. Smiley (1979) and Rothkopf (1991) present further theoretical results on multiplicative strategies. More recently, Paarsch (1992) found that Rothkopf’s 1969 multiplicative strategy model furnished the best empirical specification for explaining data on British Columbia tree planting contract auctions.
10. The mean-to-standard-deviation ratio increases smoothly and almost linearly with \( m \). When \( m = 1 \), the mean equals the standard deviation. When \( m = 10 \), it equals approximately 8.3 times the standard deviation, and when \( m = 100 \), it is about 78 times it.
11. A withdrawable bid auction might be even more attractive in markets where bidders are unable to compensate fully for the winner’s curse (see Kagel and Levin 1986).
12. Riley also shows that from the bid-taker’s perspective the second-price auction is the dominant form of the weighted price auction.

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REFERENCES


