The Welfare Cost of Signaling

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Might the resource costliness of making signals credible be low or negligible? Using a job market as an example, we build a signaling model to determine the extent to which a transfer from an applicant might replace a resource cost as an equilibrium method of achieving signal credibility. Should a firm's announcement of hiring for an open position be believed, the firm has an incentive to use a properly calibrated fee to implement a separating equilibrium. The result is robust to institutional changes, outside options, many firms or many applicants, and applicant risk aversion, though a sufficiently risk-averse applicant who is sufficiently likely to be a high type may lead to a preference for a pooling equilibrium. (*JEL* D82, J24, C72, J31)

Adverse selection becomes a concern when a party A faces a decision based on information possessed by a party B, whose utility is also affected by A's decision. That is, under what circumstances can party A rely on information communicated by party B?

Spence's 1973 paper introduced a model in which employers may use education as a screening device. In a related context, Akerlof (1970) provided perhaps the most widely taught adverse-selection example. Stiglitz (1975) discussed the concept of screening in the context of employment and education. All the above mechanisms are costly ways of solving an adverse-selection problem by creating an incentive to self-select. By contrast, in cheap-talk games (Crawford and Sobel, 1982; Chakraborty and Harbaugh, 2010; and references in the latter), where communication is privately and (probably) socially costless, information that can credibly be transmitted is limited, usually severely. This paper asks, since a sender must

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incur a cost of transmitting if the message is to be credible (for present purposes, the cost of obtaining, say, an MBA degree, is here labeled a cost of transmitting), to what extent can the cost be reduced for society by using a transfer instead of a pure resource cost?

We address this question not to explain common occurrences in markets, but to better understand the foundations of the economics of transacting under asymmetric information. To explore these foundations, suppose that a firm can credibly commit to considering only those applicants who pay an application fee that might be substantial.[1] A test that might distinguish between applicants in some aspect of their suitability could still be conducted, but only if the firm's resource costs of administering the test and evaluating the effectiveness shown are quite small, and an applicant's resource costs of preparing for and taking the test are negligible compared both to the resource costs of a usual signal and to the size of the application fee (the firm comparing a privately known threshold to the applicant's credit score, or her driving record, or her number of semesters on the Dean's list, or quickly searching Facebook or Instagram or YouTube for incidents). With this setup, the question becomes whether a suitably calibrated application fee can achieve the same types of signaling equilibria that are accomplished by calibrating the resource cost of the usual sort of signal, such as obtaining a particular level of education. That substantial application fees may not be common in labor markets of acquaintance does not bear on the relevance of this research.[2]

Two papers touch on this question, both less directly, and subject to objections. Wang (1997) introduces an employment model in which only if the firm commits to a wage schedule before the applicants pay the fee might an application-fee equilibrium be possible. Though set-of-wages, positive-application-fee equilibria may be possible below, this possibility need not require the firm to commit to a wage schedule before an applicant decides whether to pay the fee (cf. sec. II). As to using necessity of commitment to explain why no application fee is observed in reality as Wang (1997) does, no practical reason can be given why a firm could not commit

[1] Also credibly commit to not collecting application fees as a profitable activity without an appropriately compensated job waiting for the chosen applicant.

[2] Among many markets with adverse-selection issues exhibiting application fees are college admissions, scholarly journals, and nightclubs with live music.

to a schedule of multiple wages corresponding to multiple estimated productivities (indeed, this is a feature of nearly every job posting seen in the market for Ph.D. economists). Also, the pre-commitment argument is based on the assumption that firms have full control over wages. If the wage is instead determined through, say, alternating-offer bargaining,[3] it is obvious that applicants can still expect to attain some surplus, making a positive application-fee possible.

Guasch and Weiss (1981) suggest that applicants' risk aversion and an assumption that applicants do not have perfect information about themselves may prevent a positive-application-fee equilibrium. As shown below, risk aversion alone is insufficient to prevent a positive-application-fee equilibrium. The Guash/Weiss model requires the assumption that the labor-supply constraint is not binding, which is problematic: if there are more than enough high types applying, why test all of them? Where do the "extra" high-type applicants go? Firm profit maximization implies that they are not hired while applicants' expected returns show that they get paid and hired.[4]

By assumption, the firm genuinely wishes to hire someone and this is believed by the applicant (the credibility of many firms, especially prestigious ones, is in fact too valuable to risk fraudulently collecting application fees for nonexistent jobs).

Although the models use job-application settings, to varying degrees, they can apply to other contexts as well. For example, the job vacancy can easily be interpreted as a promotional opportunity within a firm. The model may extend to payment below productivity during a required internship period. Another possibility is that a firm may attempt to signal credibly the quality of a product or product line or a service by a donation to charity that it knows will be publicized at no cost to the firm.[5]

In the following model and variations, a separating equilibrium always exhibits a positive application fee. Whether there is a separating equilibrium depends entirely on the firm's incentives (or the firm's and headhunter's incentives in Section

[3] Cf. van Damme, Selten and Winter (1990) and references there.

[4] The models used in this paper are similar to those of Guasch and Weiss (1981). In fact, the one-firm, multiple-applicant case (section V) can be regarded as a simpler version of their model, while avoiding the "labor-supply constraint not binding" problem.

[5] For a context yielding several more examples, see Spence (2002).

II). Should no separating equilibrium exist, the firm's only options are not to hire, or to charge no application fee and hire any applicant without testing. With testing cost sufficiently low, separating equilibrium almost always exists.

I THE BASE MODEL

Consider a game between a profit-seeking monopsony employer and a potential applicant, both risk neutral. The applicant is either type 1 or type 2, and knows her own type. The firm does not know the type but correctly knows the distribution of types (probability $p \in (0,1)$ of being type 1; $1-p$ of being type 2). Type t worker has productivity t if working for the firm ($t = 1,2$). Both types can produce $r \in (1,2)$ at home if not hired.[6] At cost $c \geq 0$, the firm can conduct a small test, with probability $q \in (0.5,1)$ of correctly revealing the applicant's type and probability $1-q$ of being misleading, thus possibly of very little reliability. The hiring game is played as follows:

Step 1 The firm chooses its strategy $s = (w_L, w_H, f)$, where w_L is the wage offered to an applicant with test result 1, w_H is the wage offered to an applicant with test result 2, and f is the application fee.

Step 2 The potential applicant sees the wage/fee schedule s and decides whether or not to apply for the position. If she applies, she must pay the application fee.

Step 3 If the applicant has applied in step 2, she takes the test and the result is revealed for both the firm and the applicant. The applicant then decides whether to accept the wage offer fitting the test result.

For the above defined game, the firm's strategy space is $\mathbb{R}^3_+$. The applicant chooses $(App(f, w_L, w_H, t), Acc(f, w_L, w_H, t, x))$, in which t is her type and x is the

[6] This assumption is relaxed in Appendix D. A common default productivity accords with Spence's 1973 assumptions, and fits reasonably a case in which the differential productivity the firm seeks to uncover is firm-specific, or perhaps industry-specific, rather than yielding a similar productivity difference to most potential employers.

realized test result. "App" can be either "apply" or "not"; "Acc" can be either "accept" or "not".

To avoid trivialities, assume

$$c < 2 - r. \tag{1}$$

That is, the test cost cannot be so large that the firm would not make an offer to a known high type. For simplicity, also assume that the applicant accepts the offer if she is indifferent in Step 3, and that she applies if she is indifferent in Step 2. Assume, of course, that both the firm and the applicant play to maximize their expected payoff. The above specifications yield the following Theorem, proved in Appendix A.

Theorem 1 (Main Theorem)**.** *A strategy profile is a subgame-perfect equilibrium of the above defined game if and only if:*
*In step 1, the firm implements a separating equilibrium in which the potential applicant applies if and only if she is a high type, and hires anyone that applies while setting w_L, w_H and f such that [i] the firm makes an **A**cceptable **O**ffer:*

$$w_H > w_L \geq r, \tag{AO}$$

*[ii] the firm **M**aximizes **P**rofit:*

$$qw_H + (1-q)w_L - r = f. \tag{MP}$$

*In step 2, a type t (t=1 or 2) potential applicant applies if and only if [iii] the wage structure is **I**ncentive **C**ompatible:*

$$
\begin{aligned}
&q * max\{w_L, r\} + (1-q) * max\{w_H, r\} - r \geq f, \text{ (if t=1)}, \\
&(1-q) * max\{w_L, r\} + q * max\{w_H, r\} - r \geq f, \text{ (if t=2)}.
\end{aligned} \tag{IC}
$$

In step 3, the applicant accepts the offer if and only if the wage is no less than r. That is, for an applicant with test result x, accept if and only if $w_x \geq r$.

Equation (IC) has the potential applicant apply if the expected value added by applying is no less than the application fee f.

From (MP), $w_L \geq r$ and $w_H \geq r$ ensures that if the applicant applies, she is hired, at a test-dependent wage level. $w_H > w_L$ separates the value of applying for different types, in favor of the high type, given $q > 0.5$.

The fee determined by equation (MP) leads the high type to apply, though indifferent. Any higher fee prevents the high type from applying. The low type does not apply because, compared to the high type, she has a lower chance of receiving the high wage, but faces the same application fee. For given w_L, w_H satisfying (AO), $f' = (1-q)w_H + qw_L - r$ is the highest fee that induces the low type to apply; fees in the interval (f', f) reduce fee income, and lead the high type to strictly prefer applying, without otherwise affecting the outcome.

For example, setting $w_L = r$, $w_H > r$ and let f be determined by equation (MP) yields a subgame-perfect equilibrium. In such an equilibrium, both types are in their most productive positions (firm for high types and home for low types), and perfect separation is achieved without testing the low type, thus saving on testing cost. The application fee serves to make an imperfect sorting device (the test) perfect, even though the fee is purely a private cost rather than a social cost.

This welfare reassessment, that separating equilibria can be achieved by having the signal's cost be a transfer, rather than a resource cost, is robust to institutional changes. The online supplement extends the analysis to institutions in which [i] the applicant is freely considered, but only if she has paid a fee will the firm observe the test result; [ii] the fee must be paid for a positive (but possibly quite low) probability that the firm will observe the test results (and otherwise the firm does not incur a testing fee); and [iii] the fee must be paid for a positive probability that the firm will observe the test results, but with the fee refunded in the random event that the firm does not observe the results.[7]

Note that our base model is a screening game rather than a signaling game.[8] That is, the firm makes all the decisions first, and then lets nature and the applicant do all the separating, rather than observing some signals sent by the applicant, and then make decisions based on updated information about applicant type. Note also that, while the fee is an effective screening device, it cannot be made into a signaling device simply by moving the wage decision to the last step. Subgame perfection would require that the firm pay no more than r in the last stage, and as a result no applicant would pay a positive fee to apply. A variation in which the firm's wage decisions occur after a potential applicant has decided whether to pay an application fee is analyzed in the next section.

Comparison with Spence (1973): Spence's model differs from ours in two ways. His costly signal is an effort which per se serves no economic or social purpose, but is be assumed to create significantly less disutility for a high type than a low type. (The same differential capabilities which make a high type a more productive employee are assumed to yield the lower disutility of the communicative effort.) In this model, the signal cost is a transfer, and it would be untenable to assume that the high type had a significantly lower marginal utility of income. The monetary nature of the signal carries the social-welfare advantage that the money may be put to an equally productive purpose. It carries the disadvantage that paying the application fee generates the same disutility for the high type and the low type, so paying the application fee cannot by itself credibly signal type. Thus, we add to Spence's model a cheap test with an informational content that may be nearly meaningless (e.g., the fraction of the courses taken during her senior year that are numbered by her university as senior-level courses), but simply is more likely to

[7]The latter two institutions require the firm in step 1 to set two additional wage levels: that offered to an applicant who paid the fee but whose test result was not observed, and that offered to an applicant who declined to pay the fee.

[8]For a discussion of the difference between signaling and screening games, see sections 13.C and 13.D in MasColell, Green and Whinston (1995).

be passed by a high type than a low type. Now setting test-result-dependent wages so that the high type is nearly indifferent over paying the application fee separates: a lower likelihood of a passing test result leads the low type to see an insufficient expected advantage to paying the same application fee.

Spence's signal is often described as obtaining a university degree at a job-appropriate level (e.g., MBA), which if not justified as a human-capital investment, might be seen as very costly to a high type, and thus to society. Spence does not himself so limit his model; the signal in the right circumstances might be exceeding a carefully-chosen threshold on a standardized test, if attaining the threshold yields sufficiently high disutility for a low type. Thus, in some circumstances, Spence's separating equilibrium might have a high type incurring such a small cost transmitting a signal (that would have been highly costly to a low type) as to approximate "first-best": very low social costliness of the signals for types that actually transmit them. In this paper, the approximate first-best simply comes from the lack of social cost of the signal no matter how high the private cost to the signaler.

In the theorem above, the possibility of approaching a low private cost can be attained similarly to Spence's model. The f in the theorem must be positive, but can be made arbitrarily small, adjusting to $w_L = r$, lowering w_H to satisfy (IC).

II APPLICATION FEE IN A SIGNALING GAME: ADDING A THIRD PARTY

As discussed in section I, the base model's application fee cannot be shifted directly to a signaling device. Suppose there is perfect competition by firms hiring in this labor market, but an applicant can only be considered by a firm after she pays a fee to that particular firm. Once an applicant has paid the fee to a particular firm, that firm no longer faces any competition in hiring that worker, and so offers at most wage r. Any positive fee is then impossible.

This issue may be resolved by involving a third party. The applicant must pay a fee to this third party to enter the market; upon entry, all firms in this market can compete for her employment.

Consider a job market with multiple firms competing with each other, while still only having one applicant, with type assumptions as before. Now assume there is a headhunter, who holds some monopoly power in the market: firms can only hire a job applicant through the headhunter, who may demand a fee for the applicant to be available for hire.[9]

The hiring game is played in sequence as follows:

1. The headhunter sets a fee f.
2. The applicant decides whether to pay the fee to enter the market.
3. The firms quote wages.
4. If she has entered the market, the applicant chooses a firm and applies.
5. The applicant is tested, costing the firm c; she signs a waiver ceding the right to apply to or negotiate with any other firm.[10]
6. Applicant and firm learn the test results; previously set wages are offered to the applicant; she decides whether to accept or not; if she accepts, she is hired. If not, she returns home and produces r.

The waiver is a convenient way to [i] keep both w_L and w_H wage quotations relevant to applicant decisions, and [ii] prevent the applicant from applying to another firm if she tests low at the current firm. It yields the most straightforward comparison to the base model.

For a natural choice of tiebreakers, the main result extends sensibly:

Corollary 1 (Third-party Corollary)**.** *For the headhunter to set $f = 2–r–c$, firms to set (w_L, w_H) so that $w_H > w_L$ and* (MP) *and* (IC) *are satisfied, high types to apply, and the low types not to apply, constitutes a separating equilibrium.*

[9] A frequent example is a government agency that has to specify that an applicant meets certain criteria before she can be hired into a particular field or for a particular job. This model considers the agency setting the fee in excess of their cost of certification.

[10] Having the headhunter cover the testing cost out of fee revenue only yields obvious adjustments to the equilibria.

The setup and proof are in Appendix B.

Note that this result does not require a competitive industry seeking to hire a high type: suppose there is but a single firm who nonetheless can only hire an applicant who paid the fee to a headhunter; if the headhunter sets a fee as above, then the firm optimally sets wages (w_L, w_H) satisfying (AO), (MP) and (IC), generating a separating equilibrium.

III RISK-AVERSE APPLICANTS

This section returns to a single (risk-neutral) firm, to consider applicant risk aversion. Guasch and Weiss (1981) noted the obvious: a risk-averse applicant requires an expected return greater than r to accept the risk of an uncertain test result implying an uncertain wage. Intuitively, if the risk premium required is high enough, and the cost of hiring a low type is low enough, the firm may be unwilling to pay the risk premium as the cost of separating equilibrium, and hire everyone without testing instead. Assume both types of applicant have the same pattern of risk tolerance.

Under what conditions can a separating equilibrium be preserved? Instead of positing a particular risk-averse utility function, consider a wage/fee schedule and ask how high a risk premium is needed for a high type to accept. Specifically, in the base model, the firm may set the two wages arbitrarily close, the focal issue is the risk premium required if w_L is close to w_H.

Above, as is usual, the applicant is indifferent between a wage of 12 and fee of 2, and a wage of 13 and fee of 3. However, this section's analysis of risk aversion is clarified by generalizing to a utility function $U(w-f, f)$, for either type of applicant, with the usual concavity maintained via assuming $w \geq f \implies \frac{\partial U}{\partial (w-f)}(w-f, f)$ is decreasing in $w-f$ for any f.

Let $w > r$; there exists an $\varepsilon > 0$ small enough such that $w - \varepsilon \geq r$. Then the wage/fee schedule $w-\varepsilon$, $w + \frac{1-q}{q}\varepsilon$ and $f = w - r$ is a viable schedule to implement separating equilibrium in the risk-neutral case. As above, q is the probability the test correctly identifies the applicant's type. Since this involves risk, a risk-averse high type would demand a risk premium to accept such an offer; for clarity, treat the risk premium as being subtracted from f.[11]

Naturally, assume the risk premium increases with ε. Therefore the firm would prefer to offer wages as close to each other as possible.

A schedule s that makes the high type indifferent over accepting would not be accepted by the low type, who would end up receiving the low wage with a greater probability than the high type. So the firm only has to make sure that high types are indifferent in order to implement a separating equilibrium.

Let RP be a function that maps f and ε into the amount of risk premium that makes the high type indifferent. Then we have the following corollary:

Corollary 2 (Third-party Corollary)**.** $s^*(\varepsilon) = [w - \varepsilon, w + \frac{1-q}{q}\varepsilon, f - RP(f,\varepsilon)]$ *is a potential schedule in a separating equilibrium. The applicant types separate under this wage/fee schedule, provided the firm is willing.*

Note that all possible sets of wages satisfying equation (AO) can be represented by the above wage schedule, via changing ε. Since any separating equilibrium must satisfy equation (AO), the wage schedule can be represented as above. Given the above wage schedule, the fee must be $f - RP(f,\varepsilon)$, as the high type would not accept any higher fee, and the firm's profit is suboptimal for any lower fee. So:

Proposition 3. *Any separating equilibrium takes the form described in Corollary 2.*

Appendix B delves into risk aversion in more detail, finding: a minimal assumption for separation to occur, separation impossible with truly catatonic risk aversion, and how unusually risk aversion must be modeled for pooling possibly to be preferred.

IV Multiple firms competing for applicant

This section examines whether multiple firms competing for one risk-neutral hire can affect realization of separating equilibrium. If separating equilibrium is still

[11]The risk could be addressed by increasing w_H, but as a high type cannot ensure the high wage, subtracting from f is more straightforward.

achievable, it must be allowing the high type to get all the surplus, since otherwise another firm would offer a higher wage and attract the high-type worker away. On the other hand, the low type must be expected to get less than r if she were to apply. This immediately means that, any wage/fee schedule that implements separating equilibrium with multiple firms needs to separate the two types sufficiently far. Opportunities to separate with w_L closely below w_H are more restricted, perhaps preventing separating equilibrium were this section blending firm competition and applicant risk aversion.

For separation, the wage/fee schedule must satisfy:

$$w_H q + w_L(1-q) - f = 2 - c, \tag{2}$$

$$w_H(1-q) + w_L q - f < r. \tag{3}$$

Equation (2) ensures the high type's expected wage minus fee equals social surplus; equation (3) discourages the low type from applying. Subtracting (3) from (2) yields

$$(2q-1)(w_H - w_L) > 2 - c - r. \tag{4}$$

Equation (AO) is still needed to ensure hiring all that applied.

Corollary 4. *Any set of w_H, w_L and f which satisfies equations* (AO), (2) *and* (4) *yields a wage/fee schedule for a separating equilibrium.*

Details and a proof are in Appendix B.

V One firm, finitely many potential applicants

Instead of one potential applicant, suppose there are n, each is independently a low type with probability p; n and p are assumed common knowledge. The firm can only use one worker productively.[12]

Seeking a separating equilibrium, the firm has neither the desire nor the need to test all applicants. Let it adopt the strategy of testing one randomly selected applicant, hiring her if her test result is high, and otherwise hiring a second randomly selected applicant (possibly the same applicant as the first) without conducting even a second test.

This testing strategy can support a separating equilibrium. If there is no high type in the pool, no one applies and the firm receives no profit. As long as there is at least one high type, there are fee-paying applicants, the firm tests and hires someone. Therefore the firm maximizes expected payoff conditioning on at least one high type in the pool.

Let m_n be the realization of number of high types in a pool of n, w_u be the wage offered to the applicant getting high test result, and w_d be the wage offered to the applicant selected through the second random draw.[13] The firm maximizes the payoff:

$$E[2-c+m_nf-qw_u-(1-q)w_d \mid n,m\geq 1]. \tag{5}$$

Which can be simplified to (ignoring the constant 2-c):

$$E[m_n \mid n,m\geq 1]f-qw_u-(1-q)w_d. \tag{6}$$

A high type, competing with n - 1 rival potential applicants who are each a low type with probability p, though indifferent, will apply if facing a wage/fee schedule

[12]Note that number of workers being finite is important because with an infinite number of applicants, no matter how the firm sets up the hiring scheme, all applicants face a 0 chance of being hired, therefore no separation can occur.

[13]When $n=1$, this model reduces to the model in Mod 1, with w_u and w_d playing the role of w_H and w_L, respectively.

satisfying:

$$E[\frac{w_d}{m_{n-1}+1}+\frac{q(w_u-w_d)}{m_{n-1}+1}-f \mid n]=r, \tag{7}$$

or

$$f=E[\frac{1}{m_{n-1}+1} \mid n]*(qw_u+(1-q)w_d)-r. \tag{8}$$

Facing the same wage/fee schedule, a low type has the same expected payoff if randomly selected second, but a lower payoff if randomly selected first, as the probability of testing high is less. Therefore:

Corollary 5. *The firm can implement a separating equilibrium with any wage/fee schedule that satisfy* (AO) *and* (8).

Appendix B details how the base model is a special case of this model.

Adding an option to hire without testing, in a pooling equilibrium, the firm would simply hire the first of the applicants at wage r without testing. Unlike the introduction of more firms, introducing more applicants does not seem to qualitatively change the feasibility of using f and a test to create separating equilibria.

VI Discussion

The models presented yield the following conclusions:

- It is possible to use a transfer to implement a separating equilibrium; in that sense, the private cost of signaling need not be a social cost.
- Commitment to a wage by the firm is not necessary to use a transfer as a signaling device.[14]
- Applicant risk aversion alone is normally insufficient to prevent existence of a separating equilibrium. Considerable differences in home productivity across types may increase the likelihood that equilibrium requires pooling.

[14]Firms' credibility concerns can be avoided by having a centralized third party collect the application fee.

An assumption of the base model is that the test cost c being nonnegative.[15] A negative c may make it optimal for the firm to test everyone (the last inequality in case 3 of the proof may not hold if $c < 0$). For some situations, applying this model would naturally suggest a negative c. If the test represents some form of internship or other productive activity, and the fee as the reduced pay in this activity, there is a legitimate reason to claim that c can be negative, meaning the interns are producing more than the funds it took the firm to set up such a program.

Similar to the discussion about the internship, if an employee's type may be (imperfectly) revealed only after some periods of employment, the employment period before such revelation can be considered a test to determine the wage afterwards. Aside from the possibility that c is negative, there are two differences to the base model: the "test" cost c now is less for a high than for a low type, and the "test" is possibly perfect. There will still be a set of separating equilibria if c is still positive.

Are results affected if the firm has to spend money advertising jobs in order to attract applicants? Add to base model an assumption that the firm needs to incur a fixed cost in order to let the applicant be aware of the opportunity, i.e., to apply for the job. However, once paid it becomes a sunk cost, so it should not affect the firm's choice of wage and fee. It can affect the firm's choice of whether to enter the market.

Appendix C shows that if instead of having two potential applicant types, types are continuously distributed along an interval on the real line, the separating equilibria are not affected if the reservation wage remains constant across types.

Two main differences exist between Guasch and Weiss (1981) and the models in this paper so far: the models so far allow hiring of an applicant with a low test score, and a universal reservation wage across types. What will be the impact of relaxing the later restriction by introducing a variable reservation-wage based on type to these models? In Appendix D variable reservation wages are introduced to the base model. Applendix F provides proofs and extends to multiple firms or multiple applicants. It turns out that while variable reservation wages may yield a smaller set of wage/fee schedules implementing separating equilibrium (by requiring a minimum difference between w_H and w_L), firms still maintain the option to separate.

[15] Interestingly, the actual cost of the test c does not enter equation (MP) in determining the fee.

For models in which a pooling equilibrium is possibly optimal, the firm makes the same choice between separating and pooling as if the reservation wage is a constant.

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(Appendices A-D are for publishing with the text)

APPENDIX A. PROOF OF THE MAIN THEOREM

Step 3 is trivial. For step 2, the left-hand side of (IC) is the expected value of applying for $t = 1,2$, while the right-hand side is the cost of applying. It remains to show that in Step 1 the firm prefers the wage/fee schedules defined by equations (AO) and (MP) to all other schedules. In effect, the firm can decide who gets hired in Step 3 by changing w_L and w_H. Given w_L and w_H, the firm can decide who applies by changing f. Let $s^* = (w_L^*, w_H^*, f^*)$ be an arbitrary schedule satisfying (AO) and (MP). Facing s^*, a high type by assumption applies though indifferent, while a low type's expected payoff is $qw_L^* + (1-q)w_H^* - f^* < r$, preventing applying. An applicant under s^* is thus a high type. With the wage determined by the test result, the firm's expected profit is

$$(f^* + 2 - c)(1-p) - w_H^* q(1-p) - w_L^*(1-q)(1-p) = (1-p)(2-r-c). \quad (9)$$

With probability $1-p$, the potential applicant is a high type, who applies, pays the fee f^*, costs the firm c to be tested, is hired, has productivity 2, and is, in expectation, paid $w_L^*(1-q) + w_H^* q = f^* + r$ [from (MP)], attaining the strictly positive [from (1)] right-hand side of (9) .

It is trivial to dismiss as suboptimal any schedule s that [a] leads to only low types applying, [b] leads to neither type applying, or [c] leads to hiring only those who test low. Nontrivial alternatives fall into the following three cases.

Case 1: Only high types apply, only high-result applicant is hired

All such possibilities can be dealt with as if $w_H \geq r > w_L$. Then, adjusting (MP), the highest fee acceptable for a high type to apply becomes

$$f^{**} = q(w_H - r) > (1-q)(w_H - r). \quad (10)$$

The equality yields high-types applying though indifferent, the inequality low types not applying. Compared to s^*, the firm's profit has fallen by $(1-p)(1-q)(2-r)$, as high types who tested low were profitably hired in s^*. Reducing the fee to $f < f^{**}$ at best allows increasing w_H by $\frac{(f^{**}-f)}{q}$, which cannot yield an increase in expected profit, so offering no advantage. Same can be argued for increasing the fee to $f > f^{**}$.

Case 2: All types apply, all are hired

An applicant is always tested (as required in the base model). Initially assuming $w_H \geq w_L \geq r$, sets the highest acceptable fee to $f^{**} = (1-q)(w_H - r) + q(w_L - r)$. With both types hired, expected productivity is $2-p$, so expected profit is at most
$2-p-r-(1-p)(w_H-w_L)(2q-1)-c \leq 2-r-p-c$
$= (1-p)(2-r-c)+p(2-r-c)-p < (1-p)(2-r-c)$,
which is expected profits for s^*, as $r > 1$, $c \geq 0$. Next, reverse the initial assumption: $w_L \geq w_H \geq r$, the analysis corresponds:
$2-p-r-p(w_L-w_H)(2q-1)-c \leq 2-r-c-p$
$= (1-p)(2-r-c)+p(2-r-c)-p < (1-p)(2-r-c)$,
Again yielding lower expected profit than s^*.

Case 3: All types apply, only a high-result applicant is hired

Hiring only an applicant who tests high, as in case 1, it suffices to consider $w = w_H \geq r > w_L$. However, to get the low types to apply, the highest fee becomes $f^{**} = (1-q)(w_H - r)$, which has the low type apply though indifferent (and the high type strictly prefer applying). As no type offered wage w_L accepts, expected profit at f^{**} is

$$\begin{aligned} & f^{**} - c + [(1-p)q(2-w_H)] + \{p(1-q)(1-w_H)\} \\ &= (1-q)(w_H - r) - c + [(1-p)q(2-w_H)] + \{p(1-q)(1-w_H)\} \\ &= (1-2q)(1-p)w_H - c + 2(1-p)q + p(1-q) - (1-q)r. \end{aligned} \tag{11}$$

where the term in [] is productivity less wage for a high type who tests high, that in {} is the same difference for a low type who tests high, the first equality substitutes for f^{**}, the second collects terms in w_H. As $q > \frac{1}{2}$, the coefficient of w_H is negative, so expected profit is maximized at $w_H = r$, which sets $f^{**} = 0$. Substituting these values of w_H and f^{**} into the left-hand side of (11) yields

$[(1-p)q(2-r)] + \{p(1-q)(1-r)\} - c \leq [(1-p)q(2-r)] - c < (1-p)(2-r) - (1-p)c = (1-p)(2-c-r)$,

where dropping the nonpositive term in provides the weak inequality, and substituting the larger 1 for q and the smaller $(1–p)$ for 1 provides the strict inequality, again yielding lower expected profit than s^*.

Thus, an arbitrary wage/fee schedule s^* satisfying (AO) and (MP) attains a positive expected profit that exceeds all alternative schedules. Q. E. D.

Appendix B. Details and Extensions for Sections 2-5

The Headhunter Model

Lexicographic tiebreakers: as before, [i] the applicant is assumed to apply and work if indifferent. Also, [ii] if two firms quote wage offers with the same expected wage, it is convenient to break the tie by assuming the high type applies to the one quoting a higher w_H (thus a lower w_L), and the low type applies to the one quoting a higher w_L (lower w_H). This tie-breaker follows trembling-hand considerations (Selten, 1975). [iii] Across firms quoting identical wages, she randomizes equiprobably.

Of course, it does not matter to the applicant to whom she pays the fee. So if equation (IC) from section I holds, the high-type applicant continues to apply though indifferent, and the low type continues to strictly prefer not applying.

In a separating equilibrium, each firm for whom the probability of receiving an application is positive must set (w_L, w_H) so that $w_H > w_L$ and (MP) is satisfied. Despite introducing the headhunter, (MP) still makes the high type indifferent over entering the market and applying, and yields a strict preference for the low type to stay home.

Suppose the headhunter sets $f \leq 2$–r–c, the high type applies, and the low type may or may not apply. Were a given firm to face a pattern of wage offers in which every other firm offered the high type an expected wage below $2 - c$, its best response would be to offer slightly higher wages.

Thus, for the headhunter to set $f = 2$–r–c, firms to set (w_L, w_H) so that $w_H > w_L$ and (MP) and (IC) are satisfied, high type to apply, and the low type not to apply, constitutes a separating equilibrium. In such an equilibrium, the headhunter expropriates all the social surplus, and the high-type expected wage is 2–c, leaving firms with 0 profit in this labor market. The high type applies to the firm whose wage offer has the largest difference $w_H - w_L$, but the expected wage being driven up to productivity removes any incentive for other firms to deviate to attract the high type.

Are these the only equilibria? The headhunter can be shown, without doing algebra, to set too high a fee to allow a pooling equilibrium in which both types enter. In separating equilibrium, the headhunter extracts all surplus of an efficient labor market. The low type only enters if the chance of being hired justifies paying the fee, and hiring the low type reduces surplus. Wages yielding a high enough expected wage to yield low-type entry must pay some surplus to the high type, who has a greater probability of being offered w_H. So the headhunter would receive only a portion of the smaller surplus. Details are provided in Appendix G.

If application-fee revenue is used by the headhunter for some social purpose with social marginal valuation approximately dollar-for-dollar (or better), then the applicant's private cost of signaling is nearly a transfer, at most a negligible social cost.[16]

Risk-averse Applicant

First consider, for later comparison, catatonic risk-aversion: the applicant would always value a lottery at the lowest possible payoff. For this case, $RP(f, \varepsilon) =$

[16] For a model using a similar setup to discuss the status-seeking motive of charitable donations, see Glazer and Konrad (1996).

$r-(w-\varepsilon)+f=\varepsilon$,[17] the upper bound of the RP function. $RP=\varepsilon$ guarantees a payoff of at least r, even if the test result suggests a low type. So this case yields the low and high types evaluating the wage/fee schedule identically, preventing a separating equilibrium.

In less extreme cases, the firm can choose f to minimize RP. Suppose

$$\inf_{f\in(0,\infty)}\lim_{\varepsilon\to 0}RP(f,\varepsilon)=0; \tag{12}$$

this is equivalent to a continuous utility function at r. A separating equilibrium can be implemented by choosing the right f and setting ε as close to 0 as possible to pay almost no risk premium. This is different from Guasch and Weiss (1981), because in their model, an applicant with low test result is not considered for hire (equivalently, w_L is forced to be less than the reservation wage r), thus selecting w_L and w_H arbitrarily close is not an option. In addition, their model also assumes that the reservation wage varies with type, which prevents w_L and w_H from being arbitrarily close even if the restriction on w_L was relaxed. [18]

How must one model risk aversion so that the firm might prefer a pooling equilibrium? Assume a strictly positive risk premium for any distance between w_H and w_L (less extreme than catatonic risk aversion). Let

$$0<z=\inf_{f\in(0,\infty)}\lim_{\varepsilon\to 0}RP(f,\varepsilon), \tag{13}$$

i.e., for any f, applicant utility function is discontinuous at net income r.[19]

[17]The applicant is only willing to apply if the difference between the low wage and the fee is at least r, therefore $(w-\varepsilon)-(f-RP(f,\varepsilon))=r$, rearrange to get the first equation. Replace f with $w-r$ to get the second equation.

[18]The Guasch and Weiss assumption, though the reverse of Spence's common default productivity assumption, could be apt for a situation in which, without applying, a high type would be able to obtain a significantly greater wage in other industries than would a low type.

[19]Since all concave functions on real open intervals are continuous, there does not exist a real-valued utility function yielding (13).

Instead of production of 1 and 2, consider production level l for low type and h for high type. Compare firm's surplus in the separating equilibrium and in the pooling equilibrium in which the firm hires both types without testing.
The firm's surplus in separating equilibrium is

$$(1-p)(h-c-r-z), \tag{14}$$

and in pooling equilibrium is

$$p(l-r)+(1-p)(h-r). \tag{15}$$

Subtracting (15) from (14)

$$(1-p)(-c-z)-p(l-r), \tag{16}$$

or,

$$-c-z+p(c+z-l+r). \tag{17}$$

The firm only seeks a separating equilibrium if expression (17) is positive. The base model assumes $1 < r$ to give a welfare motivation to not hire the low types. With a similar assumption that $l < r$, separating equilibrium becomes more likely as p goes up (the low type becomes more likely, so hiring without testing becomes more harmful), as c or z goes down (cost of separating equilibrium becomes lower), as l goes down (the cost of hiring the low type becomes more harmful), as r goes up (hiring becomes more costly). Note that h is not in expression (16), since in both separating and pooling equilibria, high types are hired.
This analysis applies anytime a strictly positive risk premium is required. For example, if equation (12) holds, but for any reason ε cannot approach zero–that is, w_L and w_H cannot be arbitrarily close–a risk premium bounded above zero may be needed. A possible reason for ε not to approach zero is at-home productivity r differing with type. See Appendix D.

Multiple Firms

A separating equilibrium can be achieved unless there is an arrangement that can provide an expected wage minus fee for high types higher than $2-c$, while keeping the firm's return non-negative. Since the firms and the low type are already getting their reservation level, allowing the high type to get even more requires higher social surplus than separating equilibrium can attain. Since the only deviation from the full-information optimum in the separating equilibrium comes from testing the high types, if randomizing tests are disallowed, the only possible way to achieve higher surplus is by hiring everyone without testing, which can be checked by:

$$2-c \geq 2(1-p)+p = 2-p. \tag{18}$$

Separating equilibrium is not possible if p is so low that the firm can hire without testing while offering a sizable wage (close to 2), or if c is so high that testing is too costly to be justified.

As in sections I and II, even when c is 0 (costless test administration), separating equilibrium may still be achieved simply by having a positive application fee.

Multiple Applicants

Substituting (8) into the firm's expected profit (6) yields:

$$(E[m_n \mid n, m \geq 1] * E[\frac{1}{m_{n-1}+1} \mid n] - 1) * (qw_u + (1-q)w_d) - rE[m_n \mid n, m \geq 1]. \tag{19}$$

The firm does not separately care about w_u and w_d, so long as $w_d \geq r$ so that the wage offer is accepted, and $w_u > w_d$ to give high types greater incentive to apply than low types. Only their weighted sum, with test-reliability weights, enters (19). For all positive integers, the first term in parentheses in (19), $E[m_n \mid n, m \geq 1] * E[\frac{1}{m_{n-1}+1} \mid n] - 1$, equals 0 for any p.[20]

[20] An intuitive argument is provided in Appendix H.

APPENDIX C. CONTINUOUS TYPES

Is separating equilibrium robust to the applicant having continuous types?

Let the applicant's type be any real number in $[1,2]$, and type t generates output worth t if hired by the firm. The test still only produces two possible results: a high test result and a low test result. Let the test accuracy be q, $1 > q > 0.5$ as before, with $(2q-1)t+2-3q$ the probability type t attains a high test result. Thus, the probability of a high test result increases linearly with t, from $1-q$ for $t=1$ to q for $t=2$. Initially, assume r, the applicant's home production level, is the same for all types, and that the firm can only hire an applicant after she is tested. For the first part of this section, also assume there is a smallest monetary unit $0 < \delta < (2q-1)^{-1}$.

Hiring necessarily costs at least r in salary plus c for the test, so the firm has no interest in types below $r+c$, but wishes to hire types above $r+c$ if cheap enough. Observing only a high or a low test result, but not observing t, limits what is attainable.

For any wage/fee schedule s, a type t applicant's expected net wage is:

$$W(t \mid s) = [(2q-1)t+2-3q]w_H + [1-((2q-1)t-2+3q]w_L - f, \tag{20}$$

which is linear in t with slope $(2q-1)(w_H - w_L)$.

Consider a separating equilibrium where only types no less than a certain threshold apply. As (by assumption) both productivity and the chance of testing high increase linearly with type, so will the expected wage. The firm prefers to hire a higher type if and only if the slope of productivity, which is 1, exceeds the expected wage slope:

$$1 > (2q-1)(w_H - w_L). \tag{21}$$

Consider the case in which the firm chooses s that satisfies (AO) and:

$$W(r+c \mid s) = f, \tag{22}$$

$$w_H - w_L = \delta. \tag{23}$$

Equation (AO) ensures that the gain from applying increases with type, and the offer for testing low is accepted. Equation (22) simply mirrors the firm behavior specificaion of (MP): it has type $r+c$ apply though indifferent. Equation (23) (which implies (21)) minimizes the surplus paid to types above $r+c$, allowing the firm the maximum attainable surplus. Note that the distribution of types does not enter the equations characterizing separating equilibrium.

To consider pooling equilibrium, enable the firm to hire an applicant not taking the test. A pooling equilibrium is implemented if the firm hires everyone without testing at wage r. Equilibrium profit depends on the distribution of types.

A direct comparison of a separating equilibrium (where the hiring of an applicant not taking the test is disallowed) and a pooling equilibrium (where such hiring is allowed) would be unconvincing. So consider a separating equilibrium assuming any type applicant can decline to pay the fee in order to take the test, and may still be hired. Interestingly, an equilibrium schedule s leads to an interval of types applying and choosing to take the test, and the firm does not make an offer to non-test-taking types. To see this, suppose the firm hires the applicant even if test-taking is declined. In equilibrium, there must exist a type $t^* < 2$ such that [i] types $t \geq t^*$ apply, take the test, and are hired at wage w_H if testing high, w_L if testing low, and [ii] types $t < t^*$ apply, decline to be tested, and are hired at wage r. If $t^* \leq r$, the firm is better off not hiring non-test-takers, so an equilibrium requires $t^* > r$. However, in this case, all types are hired, all are paid at least r, and the firm incurs test-administration cost $(2-t^*)c > 0$, so it is strictly better off pooling than separating and hiring non-test-takers.

Therefore, the separating equilibrium specified by equations (AO) (22) and (23) still stands. The firm compares the loss of hiring types below $r+c$ at wage r (could be negative depending on distribution of types) with the cost of testing types above $r+c$, if the former is greater the firm implements a separating equilibrium, otherwise it implements the pooling equilibrium.[21]

Now discard the assumption about the smallest monetary unit δ, and the possibility of hiring a non-test-taking applicant. Suppose types are uniformly distributed be-

[21] A zero-cost test would make no qualitative difference.

tween 1 and 2, and let $r(t)$ be a continuous increasing function distributed on $[1,2]$, with $r(1) > 1$ and $r(2) < 2$, so that the firm still might profitably hire type 2, but not type 1. Also, assume the firm is again not allowed to hire an applicant not taking the test.

In figures I, II and III below, the thin line, $y(t) = t$, represents the gross productivity for each type. The dashed line shows net productivity, productivity reduced by test-administration cost c, a downward shift (0.5 in the graph). The thick line represents $r(t)$. By changing $s = (w_L, w_H, f)$, the firm can generate as $W(t \mid s)$, net expected wage, any line with nonnegative slope, obtaining the employ of any types for which W exceeds r, profiting by the height (net productivity - W).

These figures offer some interesting cases. Figure I illustrates the action the firm takes if $r(t)$ is a linear function of t, with $r(1) > 1 - c$ and $r(2) < 2 - c$. In this case, matching $W(t \mid s) = r(t)$ is the equilibrium, with the firm obtaining all the surplus (the shaded area). Figure II illustrates how a possible optimum of the firm might not involve hiring the highest types. This time $r(t)$ is flat until $t = 1.8$, and then steep. A possible W [22] is the dotted line in the figure, which yields the shaded area as surplus for the firm, while the triangle below the shaded region is applicant surplus. An interval of intermediate types is hired, while lower types are insufficiently productive, higher types overly expensive. Figure III shows a case in which c is so high that testing guarantees a loss, thus neither testing nor hiring occurs. Were hiring without testing possible, the firm can separate potential applicants without using the test or wage differentials. A flat wage can separate due to differences in reservation wage across types. In the case shown however, the firm does not wish to do so and prefers not to hire at all instead.

[22] Figure II is for illustration purpose only, and the W presented may not be optimal. For a case in which the firm optimally chooses not to hire if the potential applicant is of the highest type, consider figure I but let $r(t)$ be discontinuous at $t = 2$ and that $r(2) = 1.9$. The firm still chooses to match W with the rest of r but a type 2 potential applicant no longer applies.

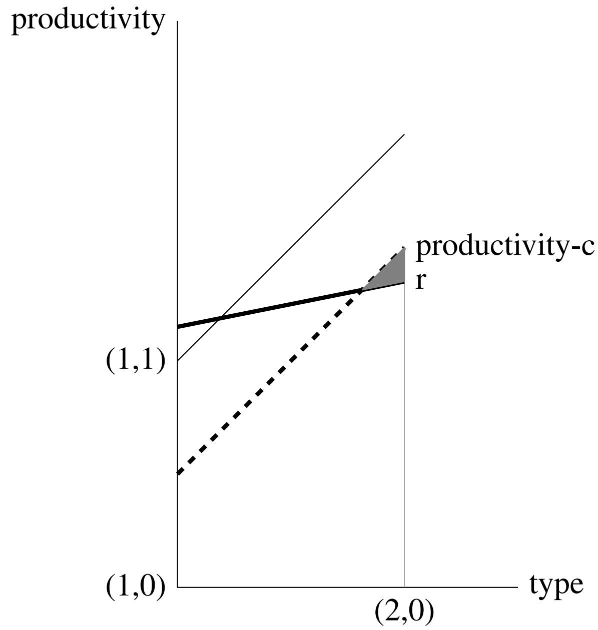

Figure I: Under linear $r(t)$, the firm maximizes profit by matching $W(t \mid s) = r(t)$.

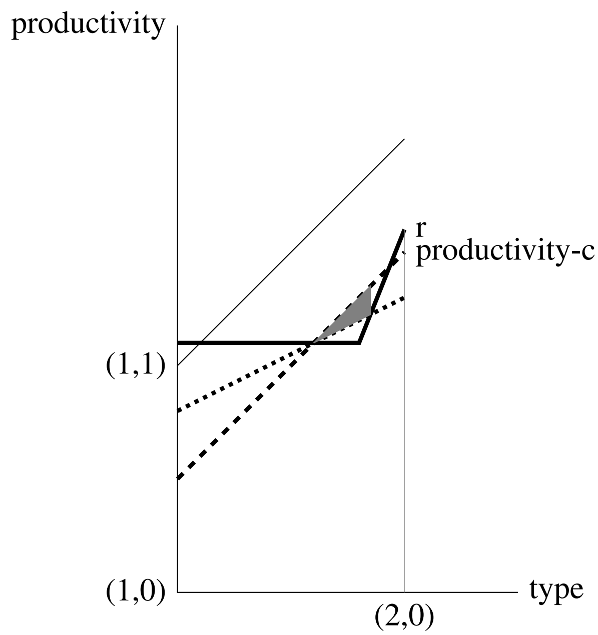

Figure II: If types are continuous and the reservation wage is variable, the firm may only hire intermediate types.

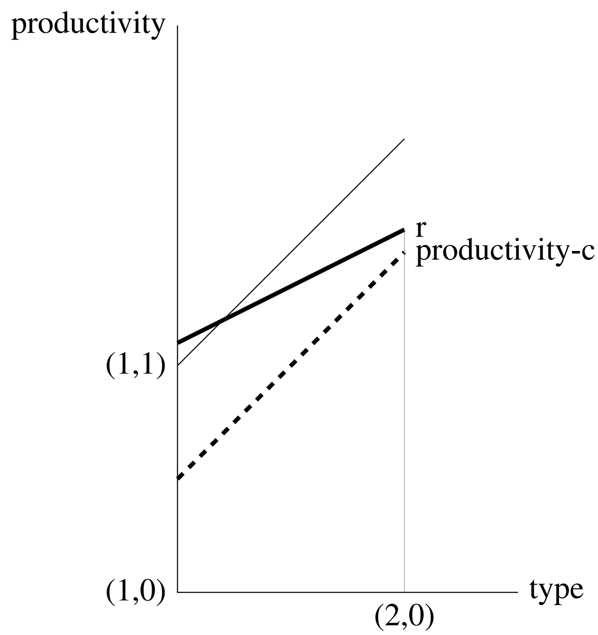

Figure III: A case in which there is no hiring nor production.

Appendix D. On Variable Reservation Wages

In the base model, instead of a universal r for both types, assume that the reservation wages of a high type potential applicant and that of a low type potential applicant are r_H and r_L, respectively. Naturally assume that $2 > r_H > r_L > 1$. Similar to the assumption given in equation (1), assume:

$$c < 2 - r_H. \tag{24}$$

Then equation (MP) is no longer sufficient to discourage a low type potential applicant from applying. In order to achieve separation, the wage/fee schedule must not only satisfy equation (AO) and (MP), but also:

$$(1-q) * w_H + q * w_L - r_L < f. \tag{25}$$

Subtract (25) from (MP) yields:

$$(2q-1)(w_H - w_L) > r_H - r_L. \tag{26}$$

Corollary 6. *A wage/fee schedule satisfying equations* (AO),(MP) *and* (26) *will implement a separating equilibrium.*

For details and a proof see Appendix F.

(The following appendices are intended to be made available in an online supplement to the paper.)

APPENDIX E. MODIFICATIONS TO THE INSTITUTION

In the base model's separating equilibrium, the firm seeks to exclude the low type, has no option but to test an applicant, and the cost c of conducting a test of limited reliability is unavoidable. That a separating equilibrium results may be no surprise. This section considers three institutions via which the firm might hire without necessarily testing.

[i] Suppose, instead of only being considered if the applicant pays the fee, she is considered automatically, but to be tested requires paying the fee. In step 1, the firm, in addition to (w_L, w_H, f), now chooses w_N, the wage for an applicant not paying the fee.
For this first modification, there can be pooling equilibria, if [a] the cost of testing is high, [b] the social loss over employing a low type is low, or [c] the probability of a low type is quite small. Specifically, if $\frac{c(1-p)}{p(r-1)} > 1$, the adverse selection problem does not justify spending resources identifying the high type.[23] It offers a fee so high or w_H and w_L so low that the applicant does not take the test and then hires both types at wage $w_N = r$ without testing. Therefore, if the above inequality is met, there is a set of pooling equilibria but no separating equilibria.

[ii] For the above modification or the base model, instead of testing everyone who paid the fee, suppose the firm gives fee payers a random chance m of actually being tested. In addition to (w_L, w_H, f), the firm also selects w_M, the wage offered if the applicant paid to take the test but was not randomly selected to take it, and w_N, the wage offered if the applicant applied but did not pay for a chance to take the test. The firm can set m as close to 0 as possible and can still implement a

[23] In a pooling equilibrium where the firm hires without testing, its profit is $p + 2(1-p) - r = 2 - p - r$, in a separating equilibrium, its profit is $(1-p)(2-c-r)$. Comparing the two: $2 - p - r > (1-p)(2-c-r) \iff c(1-p) - p(r-1) > 0 \iff \frac{c(1-p)}{p(r-1)} > 1$.

separating equilibrium. It accomplishes this by setting wage/fee schedule with $w_H > w_L \geq r > w_N$, $w_M \geq r$ and so that the high type pays the fee though indifferent, and the low type does not pay. This produces a result approaching the full-information labor allocation, while the firm extracts all the surplus. Therefore there cannot be a pooling equilibrium.

[iii] As just above, the firm always considers the applicant, and an applicant can decide to pay the fee and request to be tested, with the firm randomly administering the test with chosen probability m. Now, however, suppose the application fee is refunded unless the test is actually administered. The firm can again approach the full-information optimum, as in [ii] by setting $w_H > w_L \geq r > w_N$, $w_M \geq r$ and so that high type pays the fee though indifferent, and let m approach 0.[24]

Details of separating equilibria

Here w_M is used to denote the wage for someone who signs up for the test but not receiving a test, w_N is for someone not signing up for the test.
For [ii], to achieve separation, the firm makes the high type indifferent over paying the application fee:

$$\begin{aligned} &-f+mq(w_H)+m(1-q)(w_L)+(1-m)w_M = r \\ &> -f+m(1-q)(w_H)+mq(w_L)+(1-m)w_M. \end{aligned} \tag{27}$$

The inequality ensures that the low type does not pay the fee, and is achieved given (AO). Conditioning on separation, the firm wants to hire the fee-payer for sure, so w_L, w_H and w_M are all no less than r. Even if the firm is allowed to hire a non-fee payer, it does not wish to do so, which yields $w_N < r$. The fee achieves separation. Then firm's profit is

$$(1-p)(2+f-mq(w_H)-m(1-q)(w_L)-(1-m)w_M) = (1-p)(2-mc-r), \tag{28}$$

[24] A similar arguement to the ones in [ii] and [iii] is made by Stiglitz (1975) and mentioned in Guasch and Weiss (1981).

with equality due to the expected wage being $r+f$ [from (27)]. As m goes to 0 this approaches the full-information optimum, so a pooling equilibrium can never be more profitable (even allowing w_N as in [i]).
For [iii], the separating condition becomes:

$$\begin{aligned} &-fm+mq(w_H)+m(1-q)(w_L)+(1-m)(w_M)=r \\ &> -fm+m(1-q)(w_H)+mq(w_L)+(1-m)w_M, \end{aligned} \tag{29}$$

and the firm's profit becomes:

$$(1-p)[2+fm-mc-(r+fm)]=(1-p)(2-mc-r). \tag{30}$$

This, again, approaches the full-information optimum.

Appendix F. Details for Variable Reservation Wage

Base model: a firm can always find such a wage/fee schedule by first choosing any pair of w_H and w_L satisfying equation (AO) and with a wide enough difference to satisfy equation (26). Then calculate the fee using equation (MP). Therefore a separating equilibrium is achievable by the firm.
In order to check the optimality of such a separating equilibrium for the firm, compare the variable reservation wage model with the one if the reservation wage for both types is r_H. The separating equilibria in both cases yield the same return for the firm: the potential applicant is only tested and hired if she is a high type, and the firm claims all the surplus. Appendix A shows that in the later case, a separating equilibrium is optimal for the firm. Therefore a separating equilibrium is also optimal in the former case if there is no way for the firm to take advantage of a reduced cost to hire a low type. Indeed, in a separating equilibrium, since the firm hires only if the potential applicant is a high type and receives all the surplus, the only possibility to improve comes from hiring the potential applicant if she is a low type as well and receive surplus from a) the production of the low type or b) savings on

the cost of conducting the test. Since the low type reservation wage is still greater than her productivity, the surplus from a) is negative. As for b), the assumptions in the base model require any hired worker to take the test, preventing the firm from saving on test cost. Therefore, for this model, it is optimal for the firm to implement a separating equilibrium.

[i]: this adds the possibility of a pooling equilibrium in which the firm hires regardless of the type without testing. However, compared to the case of a single reservation wage r_H, the firm does not change its behavior under variable reservation wage. Earlier in this section, separating equilibria yield the firm the same payoff in both cases. In order to implement a pooling equilibrium, the firm needs to pay a wage of r_H regardless of the type, same in both cases. Therefore the variable types model has the same solution (either separating equilibria or pooling) as the one in which both types having a reservation wage of r_H.

Multiple firms:f the variable reservation wage assumption discussed earlier is added to the model with multiple firms, equation (3) needs to be adjusted accordingly:

$$w_H(1-q)+w_Lq-f<r_L. \tag{31}$$

Subtracting this from equation (2) yields:

$$(2q-1)(w_H-w_L)>2-c-r_L. \tag{32}$$

This is comparable to equation (4). The rest of the analysis remains the same as in Section IV. Therefore, the solution to the multiple firms competing for an applicant model with variable reservation wage is the same as if the reservation wage for both types is r_L.

Multiple applicants: apply the same variable reservation wage assumption to Section V. Equation (7) changes to

$$E[\frac{w_d}{m_{n-1}+1}+\frac{q(w_u-w_d)}{m_{n-1}+1}-f\mid n]=r_H. \tag{33}$$

And equation (8) changes to

$$f = E[\frac{1}{m_{n-1}+1} \mid n] * (qw_u + (1-q)w_d) - r_H. \tag{34}$$

In addition, to discourage low types from applying, the following must be satisfied:

$$f > E[\frac{1}{m_{n-1}+1} \mid n] * ((1-q)w_u + qw_d) - r_L. \tag{35}$$

Subtracting this from equation (34) yields:

$$E[\frac{1}{m_{n-1}+1} \mid n] * (qw_u + (1-q)w_d) - r_H > E[\frac{1}{m_{n-1}+1} \mid n] * ((1-q)w_u + qw_d) - r_L, \tag{36}$$

which can be rearranged into:

$$(2q-1)E[\frac{1}{m_{n-1}+1} \mid n] * (w_u - w_d) > r_H - r_L. \tag{37}$$

Equation (37) specifies how much of a wage difference is required in order to implement a separating equilibrium, similar to equation (26) in the base model. The firm implements a separating equilibrium by choosing a wage/fee schedule satisfying $w_u > w_d \geq r_H$, (34) and (37).

In the pooling equilibrium, the first of the applicants is hired at wage r_H. The firm chooses a separating equilibrium or the pooling equilibrium based on its expected payoff.

APPENDIX G. OTHER EQUILIBRIA WITH A THIRD PARTY

First, there is no equilibrium in which only the high type enters, but is hired only with a high test result. Were that the situation, the headhunter would get no revenue with a fee higher than $q(2–r–c)$. Competition forces the firms to zero profit, but a firm deviating to hire a low-test-result applicant at wage r attains a positive profit.

Suppose a situation in which both types enter with positive probability. Initially suppose firms 1 and 2 set wages (w_{L1}, w_{H1}) and (w_{L2}, w_{H2}), with (by labeling choice) $w_{H1} > w_{H2}$. For firm 2 to attract the high type requires $w_{L2} > \frac{q(w_{H1}-w_{H2})}{(1–q)} + w_{L1}$. For firm 2 to avoid hiring the low type requires $w_{L2} < \frac{(1–q)(w_{H1}-w_{H2})}{q} + w_{L1}$.[25] Recalling that $q > \frac{1}{2}$ is the probability the test correctly reveals type, there is no value of w_{L2} at which firm 2 is best responding to firm 1.

If both types enter, and all firms except firm 1 offer the same wage schedule (w_L, w_H) with $w_L > r$, then firm 1 gains by deviating to w_{L1} in (r, w_L) and $w_{H1} = w_H + (1–q)\frac{(w_L–w_{L1})}{q}$ (by tiebreaker [ii], firm 1 attracts the high but not the low type).

This leaves two types of candidate equilibria as follows. First, where both types enter and low scores are hired: [a]: each firm offers the same (w_L, w_H) with $w_H > w_L = r$ (so that both types accept an offer), [b]: $f = (1–q)w_H + qr–r = (1–q)(w_H–r)$ (so that the low type applies), and [c]: $2(1–p) + p–[q(1–p)–p(1–q)]w_H–[pq + (1–p)(1–q)]r–c = 0$ (which sets w_H to compete away firm profits). In the second type, both types enter and low scores are not hired: [a']: each firm offers the same (w_L, w_H) with $w_H >= r > w_L$, [b']: $f = (1–q)w_H–r$, [c']: $2q(1–p) + p(1–q)–[p(1–q) + q(1-p)]w_H–c = 0$ (for the same reasons).

Solving [c] for w_H: $w_H = \frac{2-p-c-[pq+(1-p)(1-q)]r}{p(1-q)+(1-p)q}$. Substituting into [b]:

$$
\begin{aligned}
f &= (1–q)(\frac{2–c–p–[pq + (1–p)(1–q)]r}{q(1–p) + p(1–q)}–r) \\
\implies [q(1–p) + p(1–q)]f &= (1–q)(\{2–c–p–[pq + (1–p)(1–q)]r\}–r[q(1–p) + p(1–q)]) \\
&= (1–q)\{(2–c-p)–r[pq + (1–p)(1–q) + q(1–p) + p(1–q)]\} \\
&= (1–q)[2–c-p–r(pq + 1 + pq–p–q + q–pq + p–pq)] \\
&= (1–q)(2–c–p–r)
\end{aligned}
\tag{38}
$$

[25]Tiebreaker [ii] above generates the strict inequalities.

So the maximum application fee if low types apply and low scores are hired: $f = \frac{(1-q)(2-c-p-r)}{q(1-p)+p(1-q)}$.

As mentioned in text, surplus is reduced. To see this, notice that the $(2-c-p-r)$ term is multiplied by a coefficient less than 1 [subtract the $(1-q)$ term in the numerator from the denominator: $q(1-p)+p(1-q)-(1-q) = q(1-p)-(1-p)(1-q) = (2q-1)(1-p) > 0$, so the ratio < 1]. Then, conditioning on $2-c-p-r > 0$ yields:

$$\frac{(1-q)(2-c-p-r)}{q(1-p)+p(1-q)} < 2-c-p-r < 2-c-r-(2-c-r)p = (1-p)(2-c-r), \tag{39}$$

which is the headhunter's surplus in a separating equilibrium.

Solving [c'] for w_H: $w_H = \frac{2q(1-p)+p(1-q)-c}{p(1-q)+q(1-p)}$ Substituting into [b']:

$$f = (1-q)\frac{2q(1-p)+p(1-q)-c}{p(1-q)+q(1-p)} - r, \tag{40}$$

which is the maximum application fee if low types apply and low scores are not hired. Then the headhunter's surplus in this case:

$$\begin{aligned}(1-q)\frac{2q(1-p)+p(1-q)-c}{p(1-q)+q(1-p)} - r &< 2q(1-p)+p(1-q)-c-r \\ &< 2-p-c-r \\ &< (1-p)(2-c-r)\end{aligned} \tag{41}$$

The first inequality is based on the already established $q(1-p)+p(1-q) > 1-q$; the second inequality is based on $2-p > 2q(1-p)+p(1-q)$.[26] Therefore, the equilibria described in the text are the only equilibria for this model.

[26] $[2-p]-[2q(1-p)-p(1-q)] = 2(1-p)+p-2q(1-p)-p(1-q) = 2(1-p)(1-q)+pq > 0$

APPENDIX H. INTUITIVE ARGUMENT FOR THE COEFFICIENT IN EQUATION (19) TO BE 0

Since $E[m_n \mid n] = n(1-p)$, and the probability that $m_n = 0$ is p^n, $E[m_n \mid n, m \geq 1]$ can be calculated:

$$E[m_n \mid n, m \geq 1] = \frac{n(1-p)}{1-p^n} = \frac{n}{1+p+...+p^{n-1}} \tag{42}$$

To show that $E[m_n \mid n, m \geq 1] * E[\frac{1}{m_{n-1}+1} \mid n] - 1 = 0$, having already known that $E[m_n \mid n, m \geq 1] = \frac{n}{1+p+...+p^{n-1}}$, it only remains to show that $E[\frac{1}{m_{n-1}+1} \mid n] = \frac{1+p+...+p^{n-1}}{n}$.

Suppose there are n balls lining up from left to right. One of the balls is randomly chosen and replaced by a purple ball. Then the rest are randomly painted, each with probability p being white and $1-p$ being red. The white balls are then taken away, leaving only red and purple balls in the line. One of these balls is then randomly chosen. $E[\frac{1}{m_{n-1}+1} \mid n]$ is the probability that the chosen ball is the purple one.

If instead of randomly choosing a ball in the final step, we always choose the first one on the left. The probability of the chosen ball being purple is the same as above (conditioning on there being m+1 red and purple balls, both processes yield $\frac{1}{m+1}$ probability the chosen ball being purple, for all $m \in 0, 1, ..n-1$). But the probability in the new process is equivalent to the probability that all the balls to the left of the purple ball being white. With probability $\frac{1}{n}$, the purple ball is the leftmost ball in the original line of n balls, then with probability 1 there will be no white balls to its left. With probability $\frac{1}{n}$, there will be one ball to its left, then the probability of that ball being white is p... So the probability of the chosen ball being purple is $\frac{1+p+...p^{n-1}}{n} = E[\frac{1}{m_{n-1}+1} \mid n]$. Q. E. D