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Relevance to Practice and Auction Theory: A Memorial Essay for Michael Rothkopf

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This essay assesses the state of auction theory in a particular dimension: its relevance to practice. Most auction models are more abstract than necessary. They depend on assumptions that are highly unlikely to occur in practical situations, which are often less formal and rigid. Nonetheless, we discuss several significant steps toward offering more practical advice.

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Michael H. Rothkopf (May 20, 1939–February 18, 2008), past President of INFORMS and editor of *Interfaces*, had an influential career dedicated to the concern that research should be relevant to practice. His wide-ranging contributions to operations research included scheduling theory, queuing theory, and energy economics and policy. However, his principal area of research and influence was modeling auctions and competitive bidding. As a memorial to him, it is fitting that the journal that shares his concern assess: (1) progress that has been made in addressing practical concerns of bidders, bid-takers, auction designers, and policy makers who are looking for advice about procurement or sale via auction, and (2) areas in which auction theory is still divorced from practical concerns.

We dedicate this essay to the memory of Mike Rothkopf, and to the role that he played in auction theory. While summarizing his contributions is not our direct purpose, ignoring Mike's publications would be impossible. Rothkopf and Park (2001) offer an elementary introduction to a variety of practical advice about auctions; this essay looks a bit deeper into topics, many touched on in their introduction.

Common-Value Auctions and the Winner's Curse

Theoretical modeling of auctions as competition among strategic bidders begins with Vickrey (1961). He recognizes that many sealed-bid auctions are sensibly modeled via "first-price" auction rules; they specify that bids are submitted simultaneously, and the high bidder wins and pays the amount he bid. Vickrey establishes that a "Dutch" auction, in which prices are publicly called in descending order, and the asset is sold at the first price at which a bidder affirms a willingness to pay that price, is strategically isomorphic to a first-price auction. He introduces the standard modeling of a variety of oral ascending-price auctions as an "English" auction, which assumes that all bidders are in competition as the price rises, until each publicly exits; the last still-competing bidder wins the asset at the price at which he lost his last competitor. Vickrey also introduces a new auction model, a "second-price" auction, as the sealed-bid counterpart to the English auction: bidders submit sealed bids; the high bidder wins, but the seller has committed to a price equal to the second-highest submitted bid. (Note that we will typically refer to the situation in which bidders seek to buy from an auctioning seller, but all models that we discuss also apply readily to the reverse case of bidders who are seeking to sell to an auctioning buyer.)

Two assumptions, which form the *independent-private-values* model, are that (1) each bidder is certain of the auctioned asset's value to him, and (2) the values could be modeled as independent draws from known distributions. These assumptions were sensible for Vickrey when he introduced a new field of study, but not for the several hundred far-from-seminal papers that blithely employ them. The popularity of the independent-private-values model stems solely from the mathematical ease it generates and the broad results it yields, such as Myerson's (1981) *revenue-equivalence theorem*, not from any consideration of its relevance to practice: are bidders' valuations ever independent?

Virtually every practical situation that auction theory might address is one in which (1) a bidder faces nonnegligible uncertainty about the value of the asset (e.g., the rate of the asset's economic appreciation or depreciation, its quality, its synergies with other assets, and its suitability for the intended purpose), and (2) an asset that a bidder estimates to be more valuable is also likely to be valued more highly by rival bidders. The key first steps in addressing these aspects arise in the development of the *common-value model*, which is best seen as a polar opposite of the independent-private-values model. Its primitive assumptions are that the auctioned asset has the same value to any bidder, and that no bidder knows its value with certainty. Each bidder possesses private information best viewed as an estimate of asset value.

These assumptions do not yield nearly the mathematical neatness of the independent-private-values model. However, they are quite acceptable stylizations of important markets, such as auctions of exploratory offshore oil leases, rights to airwave frequencies (e.g., for mobile communications), rights to publish celebrity-authored manuscripts, and competitions to acquire start-up or distressed corporations. Rothkopf (1969) pioneers in tackling the common-value model, and in suggesting ways in which dealing with it makes auction theory more relevant to practice. He simplifies his model by not attempting to uncover the full functional relationship between a bidder's asset-value estimate and equilibrium bid. Instead, he limits his search by constraining a bidder to selecting the best estimate-to-bid ratio. (Wilson 1969 solves for the general equilibrium bid function in the two-bidder case.) Rothkopf's multiplicative-strategy model clearly points to the most significant problem when competition and uncertainty interact. These auctions exhibit an adverse-selection problem: bidders with higher asset-value estimates tender higher bids in equilibrium, and rational bids must correct for the degree of adverse selection incumbent on the highest bid.

Capen et al. (1971) christen this problem the *winner's curse*. Ed Capen convinced his superiors at ARCO Petroleum to allow publication of the actual (decision-theoretic) algorithm that ARCO used to determine its bids on offshore oil leases. The algorithm's key feature is that it does not try to pin ARCO's estimate back into the distribution from which it was drawn via central-limit-based statistical procedures. Instead, it explicitly assumes that ARCO's estimate is the highest of N estimates, an assumption likely to be correct if the other $N-1$ bidders bid less than ARCO. Capen et al. (1941) point to the absence of profit in offshore exploration as an indication that this winner's-curse correction had been missing in offshore-lease bidding.

The winner's curse has been a persisting observation in laboratory-economics studies (Kagel et al. 1995) and in such diverse industrial settings as commercial construction, celebrity-authored publishing, airwaves auctions, and competitions to sign free-agent professional athletes. (Klemperer 2000 and Kagel and Levin 2002 discuss and cite these and other examples.)

Wilson (1977) provides the first fully game-theoretic model of common-value auctions. Milgrom and Weber (1982) extend it to a general *affiliated-values model*, which allows for the possibility that an auctioned asset has both common-value and private-values elements. For example, an investment-grade artwork might have a future resale value that is the same for all bidders, but uncertain as of the auction date. In addition, it might have different possession values for different bidders. The affiliated-values model yields several prescriptions for risk-neutral sellers, if a fixed number of bidders all make risk-neutral, symmetric equilibrium bids. Bulow and Klemperer (1996) conclude that these prescriptions, taking the form that one auction mechanism (e.g., English) yields higher expected revenue than another (first-price or second-price), are less important than the revenue impact of attracting another bidder. This result, too, cannot be considered applicable to many practical situations. Harstad (2008) points out that they assume that the additional bidder arrives exogenously; in situations in which an extra bidder is present because he expects competing to be sufficiently profitable, some degree of bidder discouragement is wise.

The recognition that the value of an asset to a bidder is stochastically related to its value to rival bidders, and thus that winning an auction creates an adverse-selection problem, is the earliest major stride in bringing practical considerations into auction modeling. Rothkopf (1969) starts this recognition.

Incentive Compatibility

The standard behavioral presumption of mainstream auction theory is that bidders (1) behave rationally, and (2) presume that their rivals and the seller behave rationally. Auction theory is the principal literature that examines the notion that diverse information is both privately held and socially useful. These two aspects have combined in the (intensive and extensive) *mechanism-design* literature, which examines incentive compatibility. A game is said to be incentive compatible if it is an equilibrium for all players to fully and truthfully reveal their private information (Hurwicz 1972). For single-asset auctions, the focal incentive-compatible auction rule is the second-price sealed-bid auction, which is generally associated with Vickrey's name. Recall that this auction's distinct feature is that the highest bidder acquires the asset, but at a price set by the highest losing bid. That is, its incentive properties stem from separating the price from the bid that the winning bidder submitted. A bidder can win an asset in a second-price auction in which the highest rival bid is \$5,000 as long as his bid exceeds \$5,000; he will win and pay \$5,000 (the highest rival bid) whether he bids \$5,001 or \$7 million (this happened for a license in a 1990 New Zealand airwaves auction: the high bidder bid \$7,000,000 and paid the second-highest bid of \$5,000). It makes no difference whether you bid b_L or $b_H > b_L$, unless the highest rival bids between b_L and b_H; therefore, being outbid below the asset's value (to you) can only result in a missed profit opportunity, while bidding above its value can only yield losses. This logic yields the conclusion that the rational bid is that price at which the bidder is indifferent between winning and losing (considering whatever information can be inferred from the presumption that this price is the highest rival bid).

The Vickrey auction (i.e., second-price auction) then attains, in theory, the outcome in which each bidder truthfully reveals his reservation price—that price at which the bidder is indifferent between winning and losing. The consequences are that (1) the asset sells for its opportunity cost, its highest value in any use other than that to which the winner puts it, and (2) this opportunity cost becomes publicly known and available to guide later resource decisions (e.g., the value of additional landing slots should a congested airport add runways and gates).

An elegant theory has extended this simple insight to more abstract problems of resource allocation in the presence of private information. In such problems, the Vickrey-Clarke-Groves (VCG) mechanism (Clarke 1971, Groves 1973, summarized in Ausubel and Milgrom 2006) is essentially the abstract extension of the Vickrey auction. It generalizes the notion of isolating the amount that an individual pays from the information that individual provided, using that information only to determine the allocation and payments made by other participants. VCG mechanisms might be applied to such situations as combinatorial auctions, assignments of tasks and compensations in a team, or the appropriate level of investment in infrastructure repair and allocation of the resulting costs.

Some difficulties that arise in attempting to use incentive-compatible mechanisms to handle practical problems are due more to the cumbersome nature of the problem than to questions of incentives; thus, they would also vex approaches that were willing to compromise on the cleanliness of incentives to reveal information. These are among the reasons that Rothkopf (2007) gives in his critique of VCG. Here we focus on problems that are more direct, and less surmountable, even in seemingly simple applications.

Placing an auction in a context in which it is not the only decision, not the only economic activity, necessarily makes incentive compatibility look substantially less attainable. Rothkopf et al. (1990) are among the first to consider some of the problems that arise. They examine a simple setting in which N bidders have independent private values for a single asset;

they analyze the impact of the need for the winning bidder to negotiate a follow-up transaction that is essential to attaining the asset value expressed in the auction, such as connecting to an electrical grid or contracting with a distribution channel. If the auction uses Vickrey rules, then the business(es) with which the winning bidder must negotiate presumably learn both the amount of the high bid and the price paid (i.e., the second-highest bid). If the high bid is truthfully submitted (i.e., a bidder j with a value v_j bid $b(v_j) = v_j$, which would be the dominant strategy absent any considerations of follow-up transactions), then follow-up negotiations are presumably hampered by the negotiating partner knowing the level of what the partner presumably considers a windfall gain—the difference between the bidder's stated value (the high bid) and the price paid for that value (the second-highest bid). In the New Zealand case, this was the difference between \$7 million and \$5,000. The authors find that a bidder j who expects this informational disadvantage to cost $C_j(v_j)$ will rationally bid $b(v_j) = v_j - C_j(v_j)$. This reduces revenue, and with it the incentives for a seller to use a Vickrey auction. More importantly, it loses the social information advantage of incentive compatibility because knowledge of the price no longer implies knowledge of the opportunity cost, the second-highest valuation. That would require knowledge of the functional form of $C_j(v_j)$.

Postauction profit opportunities, as well as costs, assail incentive compatibility. Weber (1983) considers a series of identical assets sold via sequential second-price auctions, with the winning bidder exiting further competition after each auction. In auctions before the final round, the rational bid is below the value by an amount equal to the expected profitability of the opportunity to win one of the remaining auctions. An auction in which a losing bidder finds profitability of postauction behavior to depend on the identity of the winning bidder, fails incentive compatibility because the bidder adjusts the bid away from truthful revelation (Jehiel and Moldovanu 1996). These difficulties with moving toward application of incentive compatibility in a somewhat practical setting should not be surprising. Bidders in artwork and auto auctions arrange to communicate privately to auctioneers, to prevent rival bidders from inferring the price at which they will cease competing. Such arrangements make it clear that bidders go out of their way to avoid having rivals observe their private information.

Another consideration that limits the practicality of second-price auctions in particular is the reliance of theoretical predictions on the bid-taker following the rules, and on bidder certainty that this will happen. Auction theory is generally presumptive about rules rigidity; however, situations differ in practice. Rothkopf and Harstad (1995) is a rare paper that questions bid-taker credibility. They point out that a bid-taker learns all bids, including the winner's bid, prior to announcing auction results. Hence a fictitious bid, which a bid-taker might choose to introduce, could be chosen so that the winner's bid is still high. However, the fictitious bid would set the price closer to the highest bid than would a price set by the second-highest actual bid. Suspicion of this possibility leads bidders not to treat the auction as incentive compatible. Thus, their paper explains why second-price auctions are rare (at least in the private sector and in situations in which public-sector bid-taker credibility is incomplete). In contrast, as Rothkopf and Harstad (1995) and Chakraborty and Kosmopoulou (2004) discuss, a bid-taker in an English auction must decide whether to engage in similar behavior (bid-running or using a shill bidder) in the absence of information disclosing how high the winning bidder will compete.

A large segment of the mechanism-design literature relies on the revelation principle (Gibbard 1973, Myerson 1979) to limit consideration to *direct-revelation mechanisms*. An example best illustrates this terminology. Instead of conducting a first-price, sealed-bid auction, the seller of an asset could simply instruct each bidder to submit a number that is the highest price at which that bidder would be willing to attain the asset. Simultaneous with this instruction, the seller would commit himself to a behavior of calculating (once these reservation prices had been submitted) the bids bidders would have submitted in the first-price auction, and submitting these calculated bids on the bidders' behalf. Given that these calculated bids would actually be an equilibrium of the first-price auction, then the revelation principle concludes that it is an equilibrium of the direct-revelation game for each bidder to truthfully provide his reservation price to the seller. That direct-revelation games are irrelevant to practice stems from many absurdities, ranging from trusting the seller to assuming that bidders would under any circumstances reveal their private information. Direct-revelation games are not used in practice—and will not be used.

We close this section by pointing out one more difficulty in any practical effort to develop efficiency-attaining auction mechanisms with strong incentive properties. An assumption of the standard model is the truly private nature of private information. The canonical model assumes that bidder j's private information is summarized by a random scalar X_j, drawn from some distribution $F(X_j, z)$, where z represents model-specific parameters. The model assumes that all other bidders know the functional form of F, but know nothing else about X_j. Harstad et al. (1996) point out that the more realistic assumption would be that a bidder's private information is only partly private. Bidding for offshore leases is an example. Oil exploration firms take their own seismic readings, but incorporate the cost of exploratory well rigs into valuation models; they use the same subcontractors as their competitors to construct these well rigs, and this aspect of their valuation is known to their competitors. Even more complications arise in an auction among general contractors competing for a contract to construct a skyscraper. Each bidder must come up with his own estimate of the cost of fulfilling the contract; however, bidder 1 might (for example) have obtained his electrical-components quote from the same electrical subcontractor as bidder 2, and his HVAC-components quote from the same HVAC subcontractor as bidders 3 and 4. Therefore, private information is only partially private. Mild assumptions maintain the efficient-outcome characteristic for an English auction, but a stringent assumption is needed for efficiency in a Vickrey auction. The authors provide an impossibility result for a first-price sealed-bid auction to achieve efficiency. Hence, this challenge to the practical application of auction theory is broader than merely a challenge to incentive-compatible mechanisms.

Dotage of economists (and, lately, computer scientists) on incentive-compatible mechanisms has shown little signs of waning. However, Mike Rothkopf has been a visible critic both via direct criticism (Rothkopf 2007) and the more potent demonstrations of inherent obstacles to practical application, such as follow-up transactions and partial information overlaps.

Bids as Commitments

Mainstream auction theory does not consider the possibility that, upon learning that his bid has won, a bidder might wish to back out of the commitment presumed in his bid. Harstad and Rothkopf (1995) move auction theory in a practical direction by showing that even rationally submitted bids might be winning bids that yield negative expected profit after rival bids become known. In these circumstances, the winning bidder will prefer to withdraw his bid rather than carry out its commitment. Dyer and Kagel (1996) find this to be a widespread occurrence in major building-construction auctions. Harstad and Rothkopf (1995) show that in many circumstances, it will pay a bid-taker (higher expected revenue if selling, lower expected project cost if buying) to allow winning-bid withdrawals, perhaps at a cost. In essence, if bidders are appropriately "insured" against unlimited losses that become apparent when losing bids become known, they will bid more aggressively, to the bid-taker's benefit.

Harstad and Rothkopf (1995) offer a proposal that yields more aggressive bidding—a *compensation penalty*, which guarantees that the bid-taker will be compensated for the financial impact of a bid withdrawal. For example, in a multiround ascending auction, a bidder who bids \$2 million guarantees that the seller will receive \$2 million, although not necessarily from that bidder. If the \$2 million bid is tentatively winning and is later withdrawn, should the asset sell for \$1.9 million, the bidder who has withdrawn the \$2 million bid must pay a \$100,000 compensation penalty. If, after the bid withdrawal, the bidding reaches at least \$2 million, then the withdrawal comes to be without penalty. This compensation-penalty bid-withdrawal rule was circulated in a 1993 working paper, prior to its 1995 publication; the US Federal Communications Commission (FCC) adopted it for its airwaves license auctions that began in 1994 (the FCC explicitly acknowledges the value of this suggestion in public docket 93-293). Since then, the compensation-penalty rule has been widely used in some 40 airwaves auctions in 25 nations, totaling over \$200 billion in revenue.

In some situations, such as sealed-bid auctions to fulfill an office-construction contract, the issue of a commitment to the winning bid arises only after the auction. If additional information about the cost of fulfilling the contract becomes available to the winning bidder and the bid-taker after the auction, then typically the winning bidder must either proceed (and fulfill the contract at the auction price) or default (and lose the deposit that accompanied the bid). Waehrer (1995) analyzes such a case when all parties are risk neutral; he finds that the bid-taker's expected payoff is unaffected in equilibrium by the level of the required deposit, unless renegotiation after default is either impossible or the winning bidder has no bargaining power in any such renegotiation. Given these renegotiation constraints, the bid-taker is worse off with a larger deposit requirement.

The issue of the winning bidder being committed to final usage of the asset won at auction goes largely unmentioned in the standard literature. More specifically, the winner cannot resell the asset to a losing bidder. This constraint is clearly inconsistent with the quite active secondary markets in lease contracts for oil exploration, airwaves rights, and investment-grade art, among others. Zheng (2002) considers this practical issue, albeit it in a highly rational (and highly original) model, wherein a seller anticipates the probability that a winning bidder then offers the asset up for resale to a previously losing bidder who yet again may resell it. Freeman and Freeman (1990) provide a historical example of an estate sale of an extensive private library in which a large number of such resales occurred, although they attribute much of this activity to collusion in earlier sales. Zheng finds that in settings in which bidders draw private values from asymmetric distributions, the optimal behavior by the original owner might involve a positive probability that the ultimate possessor when resales cease is not the highest-valuing bidder.

Auction theory has made progress and gained insights, in large part because it assumes commonly known and rigidly followed rules. (For example, scholars' models have taught us far less about transactions that result from unstructured negotiations.) This structure, however, is notably more rigid in theory than in practice. Mike Rothkopf and his coauthors have been prominent in this recognition and in the adjustment of models toward less rigid and more practical positions.

Backing Off from Abstractions

Models necessarily abstract from reality, hopefully to penetrate to useful illuminations. A natural part, then, of efforts to bridge gaps between theory and practice is to address particular abstract assumptions that are not met in practice. There have been many significant steps made toward practical relevance. The sampling we briefly describe here is necessarily suggestive rather than complete.

Oral ascending auctions (English auctions) are modeled as if the auctioneer called out as prices the real numbers in increasing order, an abstraction that is at least as limiting as it is understandably simplifying. Rothkopf and Harstad (1994b) back off from this assumption to examine impacts of an auctioneer's choice of which discrete bids to call. Their paper is wide-ranging in that it fits the issues that arise when discrete bid increments are an explicit part of modeling. It identifies cases in which increasing, constant, and decreasing intervals between discrete bids maximize revenue. Interestingly, increasing intervals (generally proportional or stronger) are the norm in art auctions and stamp sales; constant intervals characterize most Internet auctions; and the airwaves auctions of the FCC and other nations have utilized decreasing intervals late in the auction. Wide increments early in an auction might speed up the process; however, they could lead to a bidder other than the highest-valuing bidder winning. Rothkopf and Harstad (1994b) present conditions for which the seller's choice of intervals coincides with the economically efficient choice. Many auctions face a real and significant trade-off between interval step size and auction duration, as we present here. The FCC's "go slow" predilection has met with Congressional opposition on several occasions. Many auto and produce auctions highlight auction-duration issues. For example, in an auto auction in which several cars with their engines running are in line, the auctioneer might make a "hit the gate" call (i.e., move this car out and bring the next forward)—a warning that he is about to deploy a quick hammer to speed up the day's sales. Rothkopf studied a South Jersey produce auction in which the auctioneer saved a few seconds by tossing the receipt to be signed to the winning bidder in a razor-slit tennis ball, thus keeping duration down to about 25 seconds. Fresh-fish and fresh-flower auctions face such severe time constraints that they employ the more rapid oral descending (Dutch) auction format, which stops at the first bid. Flower auctions thus transact in under five seconds (Kambil and van Heck 1998).

The standard model of English auctions would also fit practice better if auction dynamics were less abstract. Milgrom and Weber (1982) model the auction as if all behavior were public: all bidders' exit prices become common knowledge, and exits are irrevocable. This leads to a situation in which the last two bidders infer exactly the private information of earlier exiters. They use this information to reduce asset-value uncertainty and with it winner's curse corrections. Harstad and Rothkopf (2000) offer instead a model that they call *alternating recognition* to accord better with practice. In that model, the auctioneer initially finds two bidders who are willing to affirm prices ascendingly; he then raises the price in increments, alternating between each bidder in the affirming pair and ignoring the other bidders, until one of the alternatingly recognized pair is no longer willing to affirm the latest price. At this point, the auctioneer pauses to scan the crowd to find a new affirming bidder, thus making the exit price of the no-longer-willing bidder public information. Having found a new affirming bidder and thus a new alternatingly recognized pair, he again raises the price, alternating between each in the new pair and ignoring the other bidders. The number of bidders whose exit prices become public is then stochastic, and usually some bidders' only public behavior is silence. The revenue advantage of an English auction over a second-price sealed-bid auction, which is robust and considerable in Milgrom and Weber's (1982) model, is tenuous in the alternating-recognition model. Harstad and Rothkopf (2000) also allow exits to be revocable (what auctioneer ever refused a new highest bid from a bidder just because that bidder had ceased affirming bids earlier in the auction?), but find a natural equilibrium in which they confer no advantage. Alternating recognition accords well with practice in art, auto, stamp, horse, furniture, and estate auctions. It could well be a useful stylization of manuscript auctions, auctions for professional-athlete contracts, and eBay and other online auctions.

Mainstream comparisons of auction forms prescribe usage of one form over another when it attains higher expected revenue. The frequently unstated assumption, that the seller (few papers treat the seller and the auctioneer as strategically distinct) is risk neutral, is quite reasonable in some situations. The pinball-machine company that sells four machines a week on eBay can diversify across enough auctions to be approximately risk neutral, as can the London Transport Authority, which auctions hundreds of bus routes to bidders who compete to manage service on the routes. In other cases, the size of the risk associated with an auction outcome is negligible for a sufficiently large corporation, as when Procter & Gamble calls for bids to construct an additional research facility. However, many practical situations are best modeled as exhibiting risk aversion more on the part of the bid-taker than bidders. Examples include manuscript auctions, in which a publisher can readily diversify but an author might have a sizable fraction of lifetime income at stake, and initial public offerings (IPOs), in which the investment banks bidding to manage the IPO have more diversification opportunities than the firm's pre-IPO equity holders. Waehrer et al. (1998) examine comparisons across auction forms when the entire distribution of revenue matters to the bid-taker. In circumstances in which expected revenue is the same, any risk-averse bid-taker prefers a first-price over a second-price and a second-price over an English auction. With a suitably chosen reserve price, a bid-taker prefers the first-price over all standard auctions. First-price auctions also generate higher revenues than second-price auctions when there is uncertainty about the number of participating bidders (Pekeč and Tsetlin 2008).

Standard auction models in essence treat bidders as unconstrained in bid selection. In so doing, they ignore financial constraints that are significant characteristics of many practical bidding situations. Che and Gale (1998) examine auctions with financially constrained bidders. They analyze the independent-private-values model, but allow for a correlation between a bidder's valuation of an asset and the amount of reasonable-cost funding available to contest for it (as should be the case in reasonably functioning capital markets). They find that the first-price auction generates more revenue than the second-price auction because it reduces the probability that financial constraints are binding. When constraints take the form of hard budgets on bids placed, this revenue comparison accords with the preferences of a social planner. However, their analysis implies that an all-pay auction (in which the highest bidder wins, but the seller collects all bids) would be revenue-preferred over a first-price auction; yet all-pay auctions are virtually unseen. To us, this suggests that their assumption that the same financial constraints would apply to all auction forms might well be inconsistent with the rational operation of internal capital markets.

A scholarly toolkit containing several less-abstract models surely would give auction practitioners more assistance than a one-size-fits-all model, no matter how elegant the latter is. We have described the beginnings of filling that toolkit here. Again, Mike has played a role.

Auctions in Context

McAfee and McMillan (1987), Wilson (1992), and Klemperer (1999) provide surveys of auction theory; Stark and Rothkopf (1979) attempt a comprehensive bibliography; Krishna (2002) gives some degree of literature survey in a book intended for a graduate course; and Klemperer (2004) and Milgrom (2004) include literature surveys aimed at somewhat broader audiences. Rothkopf and Harstad's (1994a) *critical essay* stands in sharp contrast because surveying the literature takes second place to illuminating it in a critical light. In the decade and a half since circulation of that essay, its authors have received communications from a couple of score of doctoral students, across four continents. Each of their dissertations stemmed directly from addressing an issue raised in that essay.

The biggest theme of that essay was that context matters, and a theory devoid of context is likely to be devoid of practical content. The above discussions of postauction bargaining, partially private information, resale, bid-taker risk aversion, and financial constraints are examples of adapting auction theory to a practical context. This section gives additional illuminating examples.

Perhaps the most critical contextual issues are dynamic concerns that fall outside a single auction. Subsequent auctions typically involve heavy bidder overlap. Examples include:

- the irregular schedule of FCC airwaves auctions (ranging from a few months apart to seven years apart),
- the fairly regular pattern of auctions to procure construction of new public-sector buildings (such as schools in areas of expanding population),
- the seven-week schedule of preferred commercial-paper auctions,
- the daily schedule of electricity auctions in many US regions (and hourly in some), and
- the continual and indeed overlapping schedule of Google's ad auctions, or the eBay situation, in which auctions of close substitutes overlap and often conclude within minutes or hours of each other.

They raise two basic questions:

- To what extent does an opportunity to bid for a close substitute in a later auction impact the appropriate aggressiveness of pursuing the asset currently being auctioned?
- To what extent does the aggressiveness of a bidder's competition in the current auction affect participation and bidding by current rivals when this bidder meets them again in subsequent auctions?

The first illustrates impacts that future auctions can have on the outcome of a current auction; the second illustrates impacts that a current auction can have on the outcomes of future auctions.

An illumination of both the strengths of game-theoretic analysis, and the difficulties of getting game-theoretic auction research to advance in practical directions, arises in response to the first of these questions, which is treated as settled. Weber (1983), as described above, reaches the answer that, in subgame-perfect equilibrium, the bid in the current auction is reduced by exactly the expected profitability of competing in later auctions. Thus, in a second-price auction, the bid is exactly the price at which the bidder is indifferent between winning and losing (the revenue-equivalence theorem extends this result to first-price auctions). The intuitive appeal of Weber's characterization seems to have nearly prevented further studies, as if intuitive appeal *per se* generated robustness.

Yet the assumptions yielding Weber's answer defy practical application: each bidder has the same value for any one of the identical assets sold sequentially, and ceases competing in later auctions upon winning once. It would be straightforward to incorporate discounting if the sequence proceeded slowly enough. However, allowing for differential impatience in any way other than assuming the highest-valuing bidders were also the most impatient seems difficult at best. The exit of winning bidders from further competition is the most important obstacle: it characterizes none of the sequential auction settings mentioned above.

Oren and Rothkopf (1975) begin to address the second issue in a model of repeated auctions that involve the same bidders and find fertile turf between decision theory and game theory. They do not treat the sequence of rival bids over the sequence of auctions as exogenously specified random variables, but rather as involving (exogenously specified) reactions by rivals to the aggressiveness of the bidder in the current auction. They consider two types of reaction functions. In the first, rival bidders respond in kind; in the second, increased aggressiveness gets a reaction in kind, but reduced aggressiveness gets no reaction. Rothkopf (1999) revisits this model in an application to daily electricity auctions.

As game theorists (less than religious, but still familiar with the arguments, and appreciative of the general forces), we consider it perilous to specify reactions exogenously, and not investigate the rationality of the specified reactions. However, game theory offers little in the way of practical alternatives. If the sequence of auctions has a known, finite end, and outside of that finite set, a bidder will have no strategic interactions with these rivals, then game-theoretic models cleanly predict no reaction: rational behavior is determined backwards from the last auction, and is independent of history. The finite end fits few practical situations: if bidders are firms in an industry, competition presumably will continue in some form, and with it strategic interaction. When repeated auctions with the same set of bidders will continue past any finite limit, game theory provides the embarrassing wealth of the folk theorem, which asserts that each of a continuum of refined equilibria is equally plausible. Folk-theorem characterizations do not offer practical advice either to bidders or auction designers.

Dynamic contexts are not limited to future auctions, of course. There is a clear relationship between bidding in airwaves auctions and cross-licensing in later, less formal bargaining markets. Harstad was a bidding consultant for a telecom firm in the first broadband airwaves auctions in 1994–1995. His client saw very different incentives in outbidding rivals who were known to be uncooperative in cross-licensing agreements than in outbidding others who had shown cross-licensing willingness and were believed interested in cross-licensing particular airwaves licenses that the client currently held. Moreover, the telecom firm's mergers and acquisitions chief urged against "spending money to outbid some other firm for a license when I can simply buy that firm," a dictum that turned out to have far broader application than we saw at the time.

A related but distinct, practical context is the impact on auctions of the frequent and broad differences between the smoothly functioning markets of economic theories and the markets around us, impacted by frictions. The Dutch auction format is instructive here. A Dutch auction starts at a price above any bidder's willingness to pay, and successively lowers the price until one bidder claims the asset; the auction ends at the claim price. Mainstream auction theory has simply treated this as isomorphic to a first-price, sealed-bid auction because the winner pays the price he bid (i.e., stopped the descending price "clock"). The principal use of Dutch auctions is in fresh flower and fish auctions; its usage is closely related to this auction form's unique ability to economize on time usage. For example, the flower auction at Naaldwijk sells over five million lots of fresh flowers, and generates typically a half-billion euros, daily. This auction is held early enough each morning so that the buyers can ship flowers throughout Europe and North America in time for same-day retail sales. To do this, it typically sells a single lot in less than five seconds, as noted above.

In some settings, the Dutch auction is run with deliberate frictions. One example is the Brooklyn electronics store that for years ran a "Dutch auction December," in which the marked price of any item was discounted 1 percent on December 1st, 2 percent on the 2nd, and so on. The store did not typically add inventory in December, leaving buyers with the same incentives as in the Naaldwijk auction: by waiting, they might obtain a lower price; the risk is that the store might sell out of the desired item. Carare and Rothkopf (2005) analyze the key difference: that waiting entails the transaction cost of revisiting the store.

An important public-policy arena in which an auction setting can provide insight is preferential treatment for disadvantaged groups. Ayres and Cramton (1996, p. 450) explain: "Civil rights advocates have implicitly conceded that affirmative action subsidies burden the public fisc—they argue instead that the social benefits of remedying past discrimination or of promoting diversity justify the cost of the government subsidies." This analysis is myopic, they suggest, because it neglects a countervailing benefit: that the public fisc may gain from the increased competitiveness of bidders who must compete against rivals who are granted preferential treatment. They examined the record of bidding in the 1994 US "regional narrowband" airwaves auctions, and argue that subsidies granted to designated bidders (e.g., businesses labeled as small, woman- and minority-owned firms, and rural telephone companies) actually increased the government's net revenue from the auction by forcing eventual winners to continue bidding when all unsubsidized rivals had exited competition.

Rothkopf et al. (2003) build a model suggesting that this was a natural development, not a fluke. Addressing the issue in a private-values model would not make sense, and preferential treatment requires analyzing asymmetric equilibria; therefore, they adapted Rothkopf's (1969) model of multiplicative strategies. For a wide range of parameters, they find in a procurement auction that subsidizing disadvantaged bidders (when the number of bidders who are not disadvantaged is less than four) lowers expected procurement cost. They also find that the economic inefficiency generated by the increased frequency with which a less-efficient bidder wins the auction because of the subsidy has a smaller efficiency cost than the efficiency cost associated with distortive taxation. Thus, they advise a public-sector agency seeking efficiency to adopt at least a small subsidy for disadvantaged bidders.

For the last one-third century of his life, Mike Rothkopf actively promoted the notion of paying attention to context as a central aspect of making auction theory relevant to practice.

Combinatorial Auctions

Most auction models treat the sale of a single asset in isolation. Multiple-asset sales are much more common. These include auctions that are simultaneous (e.g., US offshore oil leases, Australian airwaves rights, food procurement by Atlanta schools), sequential (e.g., artwork, lease-return auto auctions, livestock), or some mix of the two (e.g., manuscript auctions, frequently auctioned items on eBay, or wine auctions in which the initial winning bidder gets the right to acquire more than one case at the stated price). Typically, a bidder's overall valuation depends on the set of assets—not just the number because the assets are typically not identical—won across all related auctions. When auctioned assets complement each other (e.g., adjacent oil tracts, a free-agent pitcher prone to wildness and a free-agent catcher with quick reflexes, a set of Atlanta schools on a choice transportation route, airwaves licenses for adjacent areas with much cross-border commuting), it is impossible for bidders to properly express their valuations in any of the separate auction sales. For example, if each of five schools in western Atlanta could receive delivered food for \$1.35 per student, per day, but a single truck delivering to all five in one route would reduce costs to \$1.18, on what cost level do you base your bidding in the five auctions?

Bidders also face a risk of winning assets in some auctions while losing auctions for assets whose value is complementary. In an airwaves auction, the Chicago area license might be worth \$300 million if a bidder does not win Milwaukee but \$315 million if that bidder does win Milwaukee; the Milwaukee license might be worth \$15 million if the bidder does not win Chicago but \$50 million should Chicago be won. It is then very risky for a bidder who does not yet know the results of the Chicago auction to bid over \$15 million for Milwaukee.

These commonly found, interrelated bidder valuations, and the inability to express such valuations in the single-asset auctions on which theorists focus, lie at the core of the disconnect between auction theory and practice. Mainstream auction theory does not offer advice to bidders, or clean suggestions to bidtakers on how to develop an auction format that will let bidders express interrelated valuations.

Thankfully, in the last decade, there have been great strides in addressing practical concerns in auction modeling. The idea of combinatorial auctions, auctions in which bidders can submit all-or-nothing "package bids" for bundles of assets in which they are interested (combinations of assets, hence the name), is implicit in the VCG mechanism. Rassenti et al. (1982) propose and study experiments of more explicit and transparent (but not fully incentive-compatible) auction designs that allow package bids. It remained an esoteric academic topic until it arose in the practical context of FCC airwaves auction-design discussions in the mid 1990s. In fact, an initial result of these discussions indicated that combinatorial auctions might remain an esoteric impractical topic, mainly because of the argument that if a package bid is permitted on any subset of N assets auctioned, an auction must potentially handle $2^N - 1$ possible bids—as many as there are nonempty combinations of assets. This makes essential tasks unmanageable for practical purposes: basic communication of bidders' preferences over all combinations, determination of high bids across combinations and allocation of assets to high bidders (because every asset can be allocated only to one bidder and multiple high bidders on combinations containing a given asset should be expected). For example, Atlanta might have to decide whether to accept a \$1.28 per-student, per-day bid to supply food to schools numbered 3–10, a \$1.26 bid to supply schools 1–6, or a \$1.27 bid to supply schools 4, 8, 12, and 16; any pair of these three bids is mutually incompatible.

We can literally trace the considerable research into practical concerns with combinatorial auctions of the last decade to a remark that Mike Rothkopf made to these two authors in 1995. He said that "just because auctions where all combinations are biddable become unmanageable doesn't mean that no combinations can be permitted; there must be some manageable middle ground." Showing that there was a manageable middle ground, and characterizing borders between systems of permitted combinatorial auctions that were (or were not) manageable, led to Rothkopf et al. (1998). This has become each author's most widely cited paper.

Specifically, that paper demonstrates that the problems of determining (1) which set of mutually feasible bids attains the highest revenue, and (2) what is the lowest bid on an arbitrary combination that will move it into the set of revenue-maximizing bids, given a collection of submitted bids, are both reducible to a standard combinatorial optimization problem of determining maximum-weight set packing on a hypergraph; thus, they are NP-complete. It then goes on to point to structural properties of this optimization problem that must be understood when designing manageable combinatorial auctions, and provides several examples of economically meaningful structures of restrictions on the set of permitted package bids that are at the limits of facile implementation. For example, in March 2008, the FCC used a design in which permitted package bids follow a tree structure, which the paper included. This FCC 700 MHz auction (#73) raised more than \$19 billion in revenue.

Introducing the optimization approach from operations research and computer science to model and implement auction procedures is a pivotal development in bringing auction theory closer to practice. This is particularly important in view of the possibilities that the heavy use of electronic communications and computation, which began widespread penetration a dozen years ago, opened up. Game-theoretic models of auctions almost completely ignore these new, practical auction-implementation issues. Operations research and computer-science methods and thinking proved to be central in making auction theory relevant to the practice of auctions conducted electronically.

Rothkopf, Pekeč, and Harstad's pioneering work on combinatorial auctions has sparked interdisciplinary research. It has brought together economists, operations researchers, and computer scientists to develop practical aspects of combinatorial-auction design, and has yielded a substantial and blossoming body of academic research, which has made combinatorial auctions one of the most active research topics across several disciplines. The result has been the establishment of an interdisciplinary research field that combines microeconomic theory with optimization and computation. To a significant extent, game theorists, operations researchers, and computer scientists (and economists to a lesser extent) are adopting techniques from the other disciplines to their own toolkits. The review by Cramton et al. (2006) of the massive body of work in combinatorial auctions, special journal issues devoted to auctions conducted electronically (Geoffrion and Krishnan 2003 and Anandalingam et al. 2005), as well as generalizations of the interdisciplinary work that began with combinatorial auctions and evolved into a field of algorithmic game theory (Vazirani et al. 2007), exhibit the extraordinary impact of Mike Rothkopf's trademark themes of widening a perspective beyond standard economics and paying attention to issues that are relevant to practice.

However, it is the ability to apply and implement in practice this large body of academic work that shows the importance of combinatorial auctions and of Mike's ideas. Combinatorial auctions are widely used in procurement (Pekeč and Rothkopf 2003 and several chapters in Cramton et al. 2006 provide references). They are considered standard in allocating transportation and logistics contracts (Ledyard et al. 2002 and Sheffi 2004). In addition, they are used in a variety of other contexts from allocating contracts for school meals (Epstein et al. 2002) to auctions of airwaves licenses, such as the \$19 billion FCC auction mentioned above. One recent Franz Edelman Award winner (Metty et al. 2005) and two Edelman finalists (Hohner et al. 2003 and Sandholm et al. 2006) addressed combinatorial-auction applications. This demonstrates the substantial impact of combinatorial auctions to business practice. It is fair to say their use in practice has surpassed the state of the art in academic research on the topic. Most notable is the absence of any game-theoretic predictions about equilibrium behavior (except for occasional stylized findings in severely limited settings). The closest findings are those that characterize the VCG mechanism in the context of a combinatorial auction. However, as Rothkopf (2007) notes, VCG is hardly applicable given the valuation burden it imposes on bidders (in the worst case, they must report a valuation for each of the $2^N - 1$ combinations). On the other hand, some of the research on computational issues related to combinatorial auctions might seem detached from issues relevant to best practice. Pekeč and Rothkopf (2003, 2006) discuss how to approach combinatorial-auction design and how to resolve potential problems by careful design choices without having to apply heavy computational firepower. The general idea is to determine what is realistically implementable, despite possibly losing some otherwise appealing mathematics, and to choose a trade-off that limits the bidders' ability to express their valuations only over relevant subsets.

The influence of Mike Rothkopf's ideas and leadership in the field of combinatorial auctions continues. A current auction topic of much interest to academics is the advertisement-space auctions that Google, Yahoo, Microsoft, and others run for placement of ads next to search results, and on other Web pages (these auctions recently expanded to include advertising space in other media). This is a multibillion-dollar business of continually repeated auctions that are combinatorial in nature; bidders submit bids for combinations of key words (or other properties of the users who are entering key words, such as their location or other stored information) that would trigger a showing of the winning bidder's ad. Lahaie et al. (2007) provide a nice review of academic research on ad auctions. An emerging issue in ad-auction research is designing auctions with the understanding that bidders are budget constrained. Again, Mike Rothkopf was among the first to propose combinatorial-auction design that considers budget-constrained bidders (Pekeč and Rothkopf 2000).

In summary, Mike's contributions in the field of combinatorial auctions are a prime example of his approach to auction modeling: focusing on what is important and relevant, understanding that the devil is often in details, and providing practical solutions that are often based on an interdisciplinary approach that goes beyond the current academic-research comfort zone.

Concluding Remarks

Mike Rothkopf extended an open door to students and visitors, and a welcome mat to scholars. Without lowering his widely known standards for serious scholarship, he was constantly cajoling doctoral students, faculty, and operations researchers in corporate, public, or research organizations to build models, incorporate practical requirements, and add to our knowledge. He seemed delighted when his own work had been supplanted, and especially when questions he had posed were being addressed. The contents of this essay reflect his legacy: auction theory is far from being a poster child for the connect-to-practice struggle he considered so central; there would be far less practical advice to offer were it not for Mike's contributions, both directly and by stimulus and encouragement.

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