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Private information revelation in common-value auctions

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Abstract

When a seller has information that could help bidders to estimate asset value, a dictum of auction theory has been that all such information should be publicly announced to bidders. The possibility of privately revealing this information to one or more bidders is introduced. Seller in some circumstances may attain higher expected revenue through revealing his information privately. Examples show that the role of private revelation is more complex than simply generating bidder asymmetry.

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1. Introduction

The question of whether a seller possessing an appraisal of an asset to be auctioned (or possessing other information affiliated with asset value) should disclose this information to bidders has been central to the strategic analysis of auctions for decades. Milgrom and Weber [17] find that, out of an abstract set specifying policies for revealing information (including censored, noisy, or partial revelation), fully and publicly announcing all information a seller possesses is the

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expected-revenue-maximizing policy. This central result underlies revenue rankings between open and sealed-bid auctions, among other auction design variables.[1]

An implicit assumption in this seminal result is that a seller's only methods for disclosing information are public methods. While considerations of fairness or equity or the seller's long-term reputation may at times constrain a seller to public announcement of any information he chooses to disclose, there are precedents in practice and in auction theory for unequal treatment of bidders.[2]

As central as information disclosure results are to understanding game-theoretic forces in auctions, it makes sense to examine this assumption explicitly. Below, a seller can disclose his information either publicly to all bidders or privately to a subset of bidders. Under some circumstances, this latter option, heretofore neglected, yields higher expected revenue than any policy of public announcement. There do not appear to be simple, sweeping characterizations as to when private revelation might be preferable to public announcement in the isolated context of a single auction. The examples provided span distribution-free and independent-signal contexts, include both asymmetric and symmetric private revelation cases, and examine second- and first-price auctions, offering insights as to what relevant forces might bear on that preference. Typically, privately revealing seller's information to one bidder creates an asymmetric situation in terms of the precision of private information across bidders. These asymmetric situations as well as some of the two-bidder auctions analyzed are fraught with multiple equilibria; below, we limit revenue comparisons to instances where a strong selection argument favors a particular equilibrium.

2. The model

Throughout, a single asset with random value V is auctioned under second-price rules.[3] Information about V is summarized by

$$(\tilde{X}, \tilde{S}) = (X_1, \ldots, X_n, S_1, \ldots, S_m),$$

[1] For example, Riley [20] builds on this analysis to show that a seller can maximize the expected present value of revenue by obtaining some of the revenue via royalties, rather than all of the revenue via the end-of-auction payment by the winning bidder. Harstad and Rothkopf [9] use this principle to revenue-rank the Japanese variant of the English auction, the alternating-recognition variant, and the second-price auction.

[2] Among many examples would be home foreclosure auctions in Texas, where every bidder except the senior lien holder must pay cash, and many government procurement auctions, in which certain designated bidders (such as veterans or minority-owned firms) are subsidized. A sensible model of the tortured IPO process in the US might consider the underwriting syndicate to be bidder(s), but it is nonetheless common to share more detailed financial information with them than with other outsiders, putting them in a uniquely well-informed position. It is widely believed that many Eastern European privatization auctions have seen one bidder uniquely well informed. The principal focus of the optimal auctions literature ([18] and many following papers, see [10] for a recent survey) has been on unequal treatment of bidders; cf. also [5].

[3] For expositional simplicity, results are presented for the pure common-value model. Most results can be extended to the general symmetric model of Milgrom and Weber [17] via steps that complicate proofs in ways familiar to students of that paper.

where $N = \{1, \ldots, n\}$ is the exogeneously specified set of risk-neutral bidders, and each X_i is a signal privately observed by bidder i. The seller privately observes $\tilde{S} = (S_1, \ldots, S_m)$. The variables $(\tilde{X}, \tilde{S})$ are affiliated with V. Unless otherwise stated, the signals X_i and S_j have identical marginal distributions with a distribution function F on a common compact support $C \subset \mathbb{R}$ and a joint distribution $F_{(\tilde{X},\tilde{S})} : C^{m+n} \to \mathbb{R}_+$.

The initial case analyzed will set $m = 1$, so that the seller and any given bidder are equally well-informed, unless the seller chooses to reveal his private information to the bidder. Making the seller's information comparable allows a connection between the asymmetric equilibrium analyzed and the unique symmetric equilibrium when each of the n private signals is observed by a separate bidder and the seller announces his information publicly.

We initially analyze the "high-value" environment

$$E[V|(\tilde{X}, \tilde{S}) = (\tilde{x}, \tilde{s})] = \alpha O^{(1)}(\tilde{x}, \tilde{s}) + (1-\alpha) O^{(2)}(\tilde{x}, \tilde{s}) \quad \text{for } \alpha \in [0,1], \tag{1}$$

where $O^{(i)}(\bar{y})$ is the ith highest component of the vector $\bar{y}$.[4] In words, some convex combination of the two highest order statistics of all the information is a sufficient statistic to estimate the unknown common value.[5] Where symmetry allows focusing on bidder 1, let $Y^1 = O^{(1)}(x_2, x_3, \ldots, x_n)$, the highest private signal observed by the rivals of bidder 1 (note that this excludes the seller's signal). Let the vector $x_{-n} = (s, x_1, x_2, \ldots, x_{n-1})$, and correspondingly identify x_{-i} with the information vector absent the ith component.

Sections 6 and 7 will investigate an additive-value environment in which

$$V = X_1 + X_2 + S_1 + S_2.$$

3. Public announcement or private revelation?

Milgrom and Weber [17] have shown the symmetric equilibrium bid function when the seller publicly announces $S = s$ to be

$$b(x, s) = E[V|X_1 = x, Y^1 = x, S = s]. \tag{2}$$

For the high-value environment (Eq. (1)), this is

$$b(x,s) = \alpha E[O^{(1)}|X_1 = x, Y^1 = x, S = s] + (1-\alpha) E[O^{(2)}|X_1 = x, Y^1 = x, S = s] \tag{3}$$

$$= \alpha \max\{x, s\} + (1-\alpha)x. \tag{4}$$

[4] A simpler version of the model is based on the assumption that

$$V = \alpha O^{(1)}(S, X_1, X_2, \ldots, X_n) + (1-\alpha) O^{(2)}(S, X_1, X_2, \ldots, X_n).$$

[5] This environment is studied in [13] to analyze concentration in bidding coalitions. The $\alpha = 1$ case is considered in [4,8,14].

Consider the alternative in which the seller privately and credibly reveals $S = s$ to a single bidder, without loss of generality taken to be bidder 1.[6] It is common knowledge among the bidders that the seller has chosen this "private revelation" mechanism. The bidder privately informed is aware that he is receiving the information privately, and the inference of a bidder not receiving private information from the seller that one of his rivals is receiving it may be based on the knowledge from Theorem 8 in [17] that public announcement is preferred to withholding seller's information.

Theorem 1. *Under private revelation, the bid functions* $b^I(x_1, s) = \max[x_1, s]$, *for bidder* 1, *and* $\beta(x_i) = x_i$ *for all* $i \neq 1$ *are equilibrium components.*

The proof of a stronger result implying Theorem 1 is in Appendix A, and is independent of the stochastic structure of the signals. This flexibility is employed below to select this equilibrium for analysis, facing the possibility that other asymmetric equilibria might exist. Further support for this equilibrium, as a natural candidate for analysis, stems from the fact that it is the product of the same logic that generates bid functions which are based on the expected asset value for a bidder tying for price-setter. This general approach closely follows Mares [12], and expands to the usual $v(x, x)$ function in [15,16] if each bidder observes a scalar signal.

The high-value environment nicely isolates forces at work in comparing the advisability of public announcement vs. private revelation of seller's information. The simplest explication is for the two-bidder case, so that the lower of the two bids sets the price. If the seller's information is announced to all bidders, expected revenue is

$$R_2^{\text{Public}}(s) = E[\alpha \min\{\max(X_1, s), \max(X_2, s)\} + (1-\alpha)\min\{X_1, X_2\}].$$

If the seller's information is revealed only to bidder 1, expected revenue is

$$R_2^{\text{Pvt}}(s) = E[\min\{\max(X_1, s), X_2\}].$$

For $\alpha = 0$, the second expected revenue is larger:

$$E[\min\{X_1, X_2\}] < E[\min\{\max(X_1, s), X_2\}].$$

These two differ only in the positive probability events $\{s > X_2 > X_1\}$, in which event revenue is increased from X_1 to X_2 and the information revealed leads to bidder 1 winning, and $\{X_2 > s > X_1\}$, in which event revenue is increased from X_1 to s, while bidder 2 still wins the auction.

For particular signal distributions, a full picture can be drawn. For example, consider two bidders, and all three signals uniform on $[0, 1]$. For $\alpha < 1/4$, private revelation is revenue-superior to public announcement for all realizations s. For $\alpha > 1/2$, public announcement is preferred to private revelation of seller's information, which is in turn preferred to no revelation. For $1/4 < \alpha < 1/2$, the

[6] The credibility of a seller's claim that the information stated is actually the appraisal observed is essentially the same issue for public announcements and for private revelations.

seller maximizes expected revenue via a policy that depends on the realization of his information, choosing to announce publicly for large s, and to reveal privately for smaller s. The critical value of s is $3 - 3/[2(1-\alpha)]$. The equilibrium of Theorem 1 continues to hold for this information disclosure policy, because it is independent of the stochastic structure of the signals. In particular, a bidder who does not observe a public announcement rationally infers that s must be below the critical value, but this inference does not alter his best reply. The general comparison is:

Theorem 2. *For any n, any continuous underlying marginal distribution of signals, there is an open neighborhood of $\alpha = 0$ for which private revelation of seller's information solely to bidder* 1 *yields higher expected revenue than public announcement for any s.*

Proof. From (3), expected revenue with public announcement is (dropping the subscript 2 for general n)

$$R^{\text{Public}}(s) = E[O^{(2)}(\alpha \max[X_1, s] + (1-\alpha)X_1, \ldots, \alpha \max[X_n, s] + (1-\alpha)X_n)].$$

By Theorem 1, expected revenue with private revelation is

$$R^{Pvt}(s) = E[O^{(2)}(\max(X_1, s), X_2, \ldots, X_n)].$$

$R^{\text{Pvt}}(s)$ does not depend on α. For all continuous distributions $R^{\text{Public}}(s)$ is continuous in α and strictly lower than $R^{\text{Pvt}}(s)$ when $\alpha = 0$, implying the existence of the open neighborhood. □

Correspondingly, there is an open neighborhood of $\alpha = 1$ for which public announcement of seller's information is revenue-superior, and an open interval of values of α for which the seller prefers to reveal low signals privately, but announce high signals publicly. For any given distribution of signals, the threshold α's bounding these intervals are decreasing functions of n, as is the critical value of s in the middle region. In general, the advantage of a public announcement relative to private revelation increases with n, as it is less likely that the privately informed bidder influences the price.

Theorem 2 allows the seller to observe s before deciding whether or not to privately inform bidder 1. The alternative explicitly considered is a public announcement. It bears mention that the alternative of withholding this information is inferior for any realization of s. That is, consider a policy γ_0 in which seller withholds his information if $s \in W$, for some set of realizations $W \subset C$, and privately reveals any $s \in C \backslash W$. A superior policy is γ_1, which is to publicly announce any $s \in W$, while still privately revealing any $s \in C \backslash W$. To see this, consider γ_2, which withholds information if $s \in W$ and publicly announces any $s \in C \backslash W$, vs. γ_3, which publicly announces all $s \in C$. Revenue of γ_3 exceeds that of γ_2 ([17, Theorem 9]), as more public information is disclosed, and bidders not hearing an announcement in γ_2 infer $s \in W$. For the same reason, the excess in revenue of γ_0 over γ_1 is exactly equal to the excess of γ_3 over γ_2. Hence, it is without loss of generality to limit consideration, realization by realization, to whether to announce s publicly or reveal it privately.

A reader familiar with Campbell and Levin [3] may conjecture that the purpose of private information revelation is to create asymmetry across bidders (an "insider"). The next four sections show this is not necessarily the case.

4. Reveal to which bidder?

Recall that the proof of Theorem 1 does not depend on the stochastic structure of the underlying signals. The bid functions found remain an equilibrium without the assumption of identically distributed signals. This robustness allows the following example addressing the title of this section. Consider for simplicity the $\alpha = 0$ case of the high-value environment (Eq. (1)), for two bidders with independent signals of differing precision, and let $C \subset \mathbb{R}_+$. More precisely, assume that bidder 2 receives a signal that is more dispersive than the signal received by bidder 1: let $X_1 \sim F$ and $X_2 \sim G$, and denote this relationship by $X_1 \preccurlyeq_{cx} X_2$, or equivalently[7]

$$\int_0^s (1 - F(x))\,dx \geqslant \int_0^s (1 - G(x))\,dx \quad \text{for any } s.$$

Theorem 3. *Let bidders* 1 *and* 2 *have independent signals* X_1 *and* X_2. *Assume that bidder* 1 *is better informed than bidder* 2 *in the sense that* $X_1 \preccurlyeq_{cx} X_2$. *A seller choosing truthful private revelation over public announcement for all s prefers to reveal his information solely to bidder* 2.

Proof. Let $X_1 \sim F$ and $X_2 \sim G$, then $\max\{X_1, s\} \sim H_1$, where $H_1(x) = 0$ for $x \leqslant s$ and $H_1(x) = F(x)$ otherwise. Analogously, $\max\{X_2, s\} \sim H_2$, where $H_2(x) = 0$ for $x \leqslant s$ and $H_2(x) = G(x)$ otherwise. Therefore,

$$\min\{\max(X_1, s), X_2\} \sim M_1(x) = 1 - (1 - H_1(x))(1 - G(x)).$$

By the computations above, $M_1(x) = G(x)$ for $x \leqslant s$ and $M_1(x) = F(x) + G(x) - F(x)G(x)$ otherwise. Correspondingly,

$$\min\{\max(X_2, s), X_1\} \sim M_2(x) = 1 - (1 - H_2(x))(1 - F(x)),$$

where $M_2(x) = F(x)$ for $x \leqslant s$ and $M_2(x) = F(x) + G(x) - F(x)G(x)$ otherwise. Expected revenue for the policy of informing bidder 1 is then

$$\begin{aligned} R^1(s) &= E[\min\{\max(X_1, s), X_2\}] = \int_0^\infty (1 - M_1(x))\,dx \\ &= \int_0^s (1 - G(x))\,dx + \int_s^\infty (1 - F(x) - G(x) + F(x)G(x))\,dx. \end{aligned}$$

[7] This is the case if, for example, X_2 is a mean preserving spread of X_1.

Expected revenue for the policy of informing bidder 2 is given by

$$R^2(s) = E[\min\{\max(X_2, s), X_1\}] = \int_0^\infty (1 - M_2(x))\,dx$$
$$= \int_0^s (1 - F(x))\,dx + \int_s^\infty (1 - F(x) - G(x) + F(x)G(x))\,dx.$$

It is clear that $R^1(s) \leqslant R^2(s)$ if and only if

$$\int_0^s (1 - G(x))\,dx \leqslant \int_0^s (1 - F(x))\,dx \quad \text{for } s \text{ arbitrary},$$

which is equivalent to $X_1 \preccurlyeq_{cx} X_2$. □

As in the previous section, for small positive values of α the ranking is unaffected: it pays the seller to reveal his information privately to the bidder who will make the most use for it, who was otherwise at an informational disadvantage. This result contrasts with Campbell and Levin [3], where expected revenue in a first-price auction is higher if the bidder who already has an information advantage is shifted to a regime where his informational advantage is increased.

Notice also that the proof of Theorem 3 does not depend on any cardinal aspects of the relative precision of the signals: X_1 may be only slightly more precise than X_2, and S quite useful, in which case the seller gives bidder 2 a considerable advantage, more than the advantage bidder 1 would have had following public announcement; or X_1 may be much more precise than X_2, and all private revelation of s is doing is to lessen bidder 2's disadvantage. Combining Theorems 2 and 3, for small α, when the bidders are symmetric in the precision of their information, the seller chooses private revelation to create an asymmetry, yet when a pre-existing asymmetry has given one bidder an informational advantage, seller uses private revelation to reduce rather than exacerbate that informational advantage. However, reducing informational asymmetry is not the sole issue: for some distributions, the seller could have attained more nearly symmetric informational precision via public announcement, but preferred privately informing bidder 2.

5. Parcel seller's information?

Returning to n identically well-informed bidders, still in the high-value environment (1), suppose now $m = 2$, i.e., that the seller observes the realization of $\tilde{S} = (S_1, S_2) = (s_1, s_2)$. Among all policies of public announcement of his information, Milgrom and Weber [17] find that the seller maximizes expected revenue by fully announcing $\tilde{S}$. Equilibrium bids with public announcement (Eq. (2)) now become

$$b(x, s_1, s_2) = \alpha \max\{x, s_1, s_2\} + (1 - \alpha)\max\{x, \min(s_1, s_2)\}.$$

Two possible policies for private revelation of seller's information will be contrasted with public announcements. The first reveals the vector (s_1, s_2) to bidder 1 and nothing to the other bidders, with this revelation policy assumed common knowledge; denote this policy "Solo." The second reveals the scalar s_1 to bidder 1, the scalar s_2 to bidder 2, and nothing to the other bidders, with this revelation policy assumed common knowledge; denote this policy "Parceled." That is, the second policy "parcels out" seller's information, still via private revelation.

Comparing expected revenues conditional on seller's information:

$$R^{\text{Public}}(s_1, s_2) = E[O^{(2)}(\alpha \max(X_i, s_1, s_2) + (1-\alpha)\max(X_i, \min(s_1, s_2))],$$

$$R^{\text{Solo}}(s_1, s_2) = E[O^{(2)}(\max(X_1, s_1, s_2), X_2, \ldots, X_n)]$$

and

$$R^{\text{Parceled}}(s_1, s_2) = E[O^{(2)}(\max(X_1, s_1), \max(X_2, s_2), X_3, \ldots, X_n)].$$

It is straightforward that $R^{\text{Parceled}}(s_1, s_2) \geqslant R^{\text{Solo}}(s_1, s_2)$. For an open neighborhood around $\alpha = 0$,

$$R^{\text{Solo}}(s_1, s_2) \geqslant R^{\text{Public}}(s_1, s_2).$$

Hence, even the inferior policy that does not parcel out private revelation of seller's information may be superior to a policy of public announcement.

6. A symmetric environment

This section indicates that the possible desirability of private revelation is not due to pathologies or to the high-value environment, or simply due to the creation of an asymmetric informational advantage for the privately informed bidder. Consider a two-bidder environment in which the common value is

$$V = X_1 + X_2 + S_1 + S_2 \tag{5}$$

and the signals are independent. In particular, let X_1 and X_2 be identically distributed with a common cumulative distribution function F, and (independently) S_1 and S_2 be identically distributed with a common cumulative distribution function G. Contrast two policies: *Public announcement* (as before, indicated via *Public*) of $(S_1, S_2) = (s_1, s_2)$, vs. *symmetric private revelation* (*Parceled*) which parcels out the information to individual bidders as in the previous section, i.e., bidder 1 is informed of $S_1 = s_1$, while bidder 2 receives the information $S_2 = s_2$. The advantage of this formulation is that revenue comparisons are simplified by the uniqueness of symmetric equilibrium under each policy. The symmetric bid function following a public announcement is (from (2))

$$b(x) = E[V | X_1 = x, X_2 = x, (S_1, S_2) = (s_1, s_2)] = 2x + s_1 + s_2.$$

The symmetric bid function following private revelation is

$$\tilde{\beta}(x, s) = \beta(x + s) = E[V | X_1 + S_1 = x + s, X_2 + S_2 = x + s] = 2(x + s).$$

Denote by P^{Public} and P^{Parceled} the random variables that describe the price following a public announcement, and respectively, a private revelation

$$\begin{aligned}
&\Pr[P^{\text{Public}} \geqslant y | (S_1, S_2) = (s_1, s_2)] \\
&\quad = \Pr\left[X_1 \geqslant \frac{y - s_1 - s_2}{2}\right] \Pr\left[X_2 \geqslant \frac{y - s_1 - s_2}{2}\right] \\
&\quad = \left[1 - F\left(\frac{y - s_1 - s_2}{2}\right)\right]^2
\end{aligned}$$

and

$$\begin{aligned}
&\Pr[P^{\text{Parceled}} \geqslant y | (S_1, S_2) = (s_1, s_2)] \\
&\quad = \Pr\left[X_1 \geqslant \frac{y}{2} - s_1\right] \Pr[X_2 \geqslant \frac{y}{2} - s_2] \\
&\quad = \left[1 - F\left(\frac{y}{2} - s_1\right)\right] \left[1 - F\left(\frac{y}{2} - s_2\right)\right].
\end{aligned}$$

Observe that if the survival function $1 - F(x)$ is strictly log-convex then

$$\Pr[P^{\text{Parceled}} \geqslant y | (S_1, S_2) = (s_1, s_2)] \geqslant \Pr[P^{\text{Public}} \geqslant y | (S_1, S_2) = (s_1, s_2)],$$

while the log-concavity of $1 - F(x)$ reverses the inequality. In conclusion,

Theorem 4. *For environment* (5), *the revenue from private revelation stochastically dominates the revenue from public announcement for every possible realization of the sellers' information* (S_1, S_2) *if and only if the survival function of private signals satisfies log-convexity. If the survival function is log-concave, the relationship is reversed.*

As with results in Sections 3–5, this theorem applies once seller has observed the realization(s) of his information, but implies that an ex ante commitment to a policy of private revelation attains higher expected revenue than an ex ante commitment to public announcement.

This theorem contrasts with the common interpretation of Milgrom and Weber's "honesty is the best policy" claim.[8] It has also a deeper interpretation since it can be noted (as in Campbell and Levin [4]), that in an additive-value auction "virtual valuations" line up with private signals if and only if the distribution is log-concave.[9]

[8] The quote is from Milgrom and Weber [17, p. 1096]. Their Theorem 9 states "In the second-price auction, no reporting policy leads to a higher expected price than always reporting X_0," while Theorem 8 had already established that "revealing information publicly raises revenues." They "formulate the seller's report very generally as $X_0' = r(X_0, Z)$, i.e., the seller's report may depend both on his information and the spin of a roulette wheel" (p. 1103). This class of information disclosure policies is not general enough to incorporate the possibility analyzed in Theorem 4, although that yields a fully symmetric situation and symmetric equilibrium. In the additive environment, the sum $s_1 + s_2$ is a sufficient statistic for (s_1, s_2), yet parceling out s_1 and s_2 is a better policy than "public honesty" when $1 - F(x)$ is log-convex. So while Proposition 4 is not a counterexample to [17, Theorem 9], it is a direct challenge to its generality.

[9] Campbell and Levin's [4] work builds on Myerson's [18] and Bulow and Klemperer's [2] mechanism design analysis, identifying the optimal mechanism in those situations where the asset is allocated to the bidder with the highest virtual valuation. Myerson's definition of virtual valuations is limited to the case

By the virtual valuation of bidder i is meant

$$V_i = V(x_i, x_{-i}) - \frac{1 - F(x_i)}{f(x_i)} \frac{\partial}{\partial x_i} V(x_i, x_{-i}),$$

which in the additive-value environment with public disclosure is

$$V_i = s_1 + s_2 + \sum x_j - \left(\frac{1 - F(x_i)}{f(x_i)} \right).$$

So, if the survival function is log-convex, then virtual valuations are decreasing in signals. Revenue-maximizing sellers should therefore seek to allocate the asset away from the highest bidder. This is exactly what is being achieved in the example above. If the seller informs the bidders publicly, the overall allocation rule is preserved: the highest signal holder attains the asset with probability one. By privately informing the bidders, the seller can change the allocation at least for some combinations of signals. This leads to the results in Theorem 4. In contrast, in Section 2's high-value environment, $\alpha = 0$ is the regular case, while for $\alpha = 1$ the smallest virtual valuation is associated with the highest private signal. Nevertheless, Theorem 2 shows that public announcement outperforms private revelation only in the irregular case. This analysis shows that regularity, understood as alignment of signals with virtual valuations, cannot be a characterization for the optimality of an information disclosure policy.[10]

Another issue that can be analyzed within the additive environment is the optimality of fully revealing private revelations. A main implication of the linkage principle for information revelation policies is that in the class of public announcements truthful disclosure revenue dominates any noisy or partial revelation.

For private revelation, the comparison of noisy to full revelation is more subtle. As mentioned above, in their symmetric two-bidder model with a binary distribution of the common value and of private information, Campbell and Levin [3] calculate expected revenue in a first-price auction under different information structures. They find it can be revenue-enhancing for the seller to reveal fully, rather than reveal partially, information which creates an asymmetric advantage;[11] in contrast, additive-environment revenue comparisons here do not depend on creating any informational asymmetry. Also, Appendix B provides a simple, additive-environment example that negates this intuition as a generalization: for a second-price

(*footnote continued*)
where values are private, or "revision effects" are additive; the definition of virtual valuations used here is that in [4]. In Bulow and Klemperer [2] the condition that virtual valuations and signals are aligned is called regularity.

[10] Curvature of the log of the survival function has a history in the auction theory literature; see [18]. Bikhchandani and Riley [1] show that log-concavity of the private signals is a sufficient condition for symmetric equilibrium to revenue outperform any asymmetric equilibrium in a two-bidder second-price common-value auction.

[11] The information revealed by seller is bidder 1's private information; as such it makes no difference in their model whether this information is publicly announced or (it is common knowledge that it) is privately revealed to bidder 2.

auction, a private revelation policy of symmetric noisy parceling generates higher expected revenue than parceling noiselessly.[12]

7. Private revelation in first-price auctions

This additive environment (5) also offers an opportunity to investigate first-price auctions.[13] Compare the revenues generated by the first-price symmetric bidding equilibria under (i) seller precommitting to a policy of public announcement, and (ii) precommitting to a policy of parceled private revelation. As before, let the cumulative distribution function of a signal be F, but let the cumulative distribution function of $X_1 + S_1$ be G. Using Milgrom and Weber [17], we can easily establish that first-price equilibrium bid functions for the two policies are

$$\Phi^{\text{Public}}(x; s_1, s_2) = 2E[X|X \leqslant x] + s_1 + s_2$$

and

$$\Phi^{\text{Parceled}}(x_i, s_i) = 2E[X + S|X + S \leqslant x_i + s_i].$$

Denote the key terms in these equations by $H(x) = E[X|X \leqslant x]$, and $\tilde{H}(t) = E[X + S|X + S \leqslant t]$. It is immediately clear that both H and $\tilde{H}$ are increasing functions. Expected revenue turns on comparison of the following two terms:

$$R^{\text{Public}}(s_1, s_2) = 2E[H(\max(X_1, X_2)] + s_1 + s_2$$

and

$$R^{\text{Parceled}}(s_1, s_2) = 2E[\tilde{H}(\max(X_1 + s_1, X_2 + s_2))].$$

Some properties of H and $\tilde{H}$ are illuminating. Appendix C proves:

Lemma 5. *If the support of S is contained in $\mathbb{R}_+$, and the distribution of X is log-concave, then $H(x) \leqslant \tilde{H}(x)$.*

This result implies that

$$R^{\text{Public}}(0, 0) < R^{\text{Parceled}}(0, 0)$$

and extends the comparison for small values of the signals s_1 and s_2. The reversed inequality can be shown for high values of s_1 and s_2. This fact is quite intuitive. Assume for simplicity that the support of S is $[0, \infty)$, and that both X and S have a

[12] This example offers a comparison within the class of private revelation mechanisms. It should be noted that for the chosen specification public announcements dominate private revelation based on parceling.

[13] We are grateful to an anonymous referee for suggesting this line of inquiry.

finite first moment. Observe that $\lim_{s\to\infty} H(s) = E[X+S]$, and hence

$$\lim_{s\to\infty} R^{\text{Parceled}}(s,s) = 2E[X+S] < \lim_{s\to\infty} 2E[H(\max(X_1,X_2)] + 2s = \lim_{s\to\infty} R^{\text{Public}}(s,s).$$

In a first-price auction, then, the seller would prefer public announcement for high values of his signals, while for low values he would choose private revelation. This contrasts with second-price auctions for the following reason: for a given estimate of asset value conditional on his observed and inferred information, a second-price bidder does not change his bid if he expects rivals to bid more competitively. In contrast, a first-price bidder, under the identical conditioning on expected asset value given all information, does rationally bid closer to his asset value estimate if he expects more aggressive competition. High values of seller's appraisals raise asset value estimates in both auction forms, but also increase bidder aggressiveness in first-price auctions.

Notice that, in essence, these results would put the seller in a position of precommitting to public announcement, but regretting this precommitment if his signals turned out to be low, or else precommitting to privately parceling out the information, but regretting this if his signals turned out to be high. However, the equilibrium formulae above are clearly not distribution-free, so a policy of not precommitting, but privately revealing low signals and publicly announcing high signals would lead bidders to change their behavior so as to incorporate this inference (a private revelation implies an upper bound on the rival's private revelation) that obfuscates any revenue comparison.

The information disclosure policy the seller should commit to ex ante, i.e., before he has received his signals, can be found by relying on the symmetry of the additive environment, which puts it within the class of models where revenue equivalence still applies.[14] At the ex ante level, if in one auction type a seller would prefer private revelation to public announcement, then it has to be true also for the other. Since under the assumptions of Theorem 4, private revelation is revenue-superior to public announcement at the *interim* stage (after observing (s_1, s_2)), it is also superior ex ante. Under those assumptions, a first-price seller should commit to private revelation rather than to public announcement.

8. Concluding remarks

Useful examples have suggested that it is the privacy of a bidder's information, not its quality, that is the source of a bidder's profit [3,6,17]. Bidders may tend to bid more sensitively with respect to privately held rather than publicly announced information. A seller may be able to reveal his information privately and credibly to

[14] See, for example, Waehrer et al. [22]: this environment satisfies their symmetry and independence requirements. Note that revenue equivalence can only be applied from an ex ante point of view. *Interim*, as indicated, first- and second-price revenues diverge.

a single bidder, and gain from that bidder's proprietary use of the information. In the environments above, when the highest estimate of asset value is not of overriding importance, this advantage of making seller's information property of a single bidder more than surmounts the benefit of a public announcement. Public announcement has a strictly higher probability of linking the price to seller's information, but a smaller impact, and proprietary usage of this information impacts revenue even in some events where s is not directly linked to the price.

When bidders are already asymmetric in the precision of their private information, Section 4's result finds greater impact of making s proprietary information by using it to reduce the asymmetry, revealing it to the less-well-informed bidder. However, in Section 5 it pays to parcel out information, still each piece to a proprietary recipient, rather than providing it all to a single bidder. That result might be interpreted similarly, by a hypothetical sequential information disclosure policy: having revealed s_1 to bidder 1 (chosen at random), bidder 1 is now at an informational advantage, which means that bidder 2 will adjust his bid more sensitively to the revelation of s_2 than bidder 1 will, so revelation of s_2 again is used to reduce (if two bidders, eliminate) informational asymmetry.

Given this environment, for a range of parameters, the seller prefers to reveal low appraisals privately, while announcing high appraisals publicly. When only the two highest signals affect asset value, the conditioning bidders infer from the lack of a public announcement—that the seller's appraisal must have been below some threshold—does not affect bids. In general, given the asymmetry generated by private revelation, such conditioning by uninformed bidders may create serious difficulties, for the resultant revenue performance of such partial public revelation via the fact of private revelation, if not for uncovering the equilibrium itself.[15] Interestingly, the equilibrium in Section 6 is also immune to this concern: a bidder aware that seller publicly announces if and only if $(s_1, s_2) \in A$, and learns privately a single s, rationally infers $(s_1, s_2) \notin A$, but this inference does not change his best reply.

The examples in Sections 6 and 7 show a more complex link between the specification of the auction model and optimal information disclosure policies. Public announcements revenue dominate private revelation only under very definite circumstances. In second-price auctions, sufficient conditions are found under which the seller would choose a private revelation policy for any set of observed signals. An example shows that fully truthful private revelation is not always revenue-maximal in the class of private revelation policies. In first-price auctions the results are less clear cut: for certain signals the seller would still choose to announce publicly rather than reveal privately. Nevertheless, we identify conditions under which a seller that can commit to a disclosure policy before observing his information would choose private revelation rather than public announcement.

Milgrom and Weber [17] argue that linking the price paid at auction to as many variables affiliated with asset value as possible, quite generally increases expected revenue. While revenue comparisons inconsistent with this "Linkage principle" have

[15] In essence, such a situation amounts to informing one bidder what s is, and implicitly informing the other bidders that s lies in a strict subset of its support (presumably a "bad news" subset).

been found in several settings,[16] this paper focuses on a dimension directly challenging only its presumed information-policy implications. In formulating seller's information disclosure options more generally than they have, we allow for the information to have a broader range of impacts, which can be thought of as linking seller's information to the price in ways more subtle, less direct, but sometimes more powerful.

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Appendix A

The equilibrium posited for private revelation in the average-value auction of Section 6 is the unique symmetric equilibrium, and as such a natural candidate for study. However, the information structure in Sections 3 and 4 is asymmetric under private revelation, and hence no symmetric equilibrium can exist. Asymmetric equilibria other than the one studied are likely to exist (and indeed, one is shown below for $\alpha = 0$ case). This appendix argues that the natural equilibrium to focus on is the equilibrium of Theorem 1. The notion of a partition-proof equilibrium (cf. [12,13]) is introduced, and the posited equilibrium is found to be the unique partition-proof equilibrium.

Let $(V, X_1, \ldots, X_n)$ be affiliated, and the X_i's exchangeable. Let there be l bidders, with the n signals partitioned among them. A *partition-proof equilibrium* is a profile $b^* = (b_1^*, \ldots, b_{n-1}^*)$, with each $b_k^* : R^k \to R$, defined independently of l, having the property that it is an equilibrium for any bidder observing k signals to bid component function $b_k^*(x_1, \ldots, x_k)$ if rivals also employ their component functions of b^*, without regard for the number of rivals or the way in which the $n - k$ signals are partitioned among them. Thus, for example, if bidder 1 observes 3 of $n = 13$ signals, $b_3^*(x_1, \ldots, x_3)$ is a best response whether facing 10 rivals each observing one signal and bidding $b_1^*(x_1)$, one rival bidding $b_{10}^*(x_1, \ldots, x_{10})$, or three rivals bidding $b_2^*(x_1, x_2)$, $b_5^*(x_3, \ldots, x_7)$, and $b_3^*(x_8, \ldots, x_{10})$, respectively. Thus, the bid functions depend only on one's own information set and are unaffected by the overall information profile. A partition-proof equilibrium is a strong requirement, but if met, yields a natural candidate for equilibrium selection, especially in an

[16] See Perry and Reny [19] for a multi-object example. Also, Harstad and Bordley [7] provide cases where a "curtailed oral auction" increases expected revenue via stopping an English auction when $k > 1$ bidders are still competing, and awarding the asset at random among them, at this last prior exit price. That auction deliberately reduces the number of informational variables to which the price is linked.

environment where a bidder may not be certain that rivals are formulating strategies independently and noncooperatively.

As in Sections 2–5, assume a second-price, common-value auction for which

$$E[V|X_1 = x_1, \ldots, X_n = x_n] = \alpha O^{(1)}(x_1, \ldots, x_n) + (1-\alpha)O^{(2)}(x_1, \ldots, x_n).$$

(Notice that this specification does not satisfy the Bikhchandani and Riley [1] sufficiency condition for the uniqueness of the symmetric equilibrium among all equilibria with increasing bid functions.) No other assumption about the stochastic structure of the vector $(X_1, \ldots, X_n)$ is made. Consider l bidders who observe disjoint subsets of this collection of dimension $k_1, k_2, \ldots$ and k_l, respectively.

Theorem 6. *The profile b^* defined by $b^*_k(x_1, \ldots, x_k) = \max\{x_1, \ldots, x_k\}$ is a partition-proof equilibrium.*

Proof. Let $\tilde{x} = (x_1, \ldots, x_n)$. Consider the problem faced by bidder 1

$$B_1(x_1, \ldots, x_{k_1}) \in \arg\max_B \; E[(V - Y_{k_1})\mathbf{1}(B \geqslant Y_{k_1})|X_1 = x_1, \ldots, X_{k_1} = x_{k_1}],$$

where $Y_{k_1} = \max\{X_{k_1+1}, \ldots, X_n\}$. The problem can be equivalently rewritten as

$$B_1(x_1, \ldots, x_{k_1}) \in \arg\max_B \int_{-\infty}^{B} (\alpha\Omega^{(1)}(\tilde{x}) + (1-\alpha)\Omega^{(2)}(\tilde{x}) - y_{k_1})\, dF(y_{k_1}|x_1, \ldots, x_{k_1}).$$

The main argument is that

$$\alpha\Omega^{(1)}(x_1, \ldots, x_n) + (1-\alpha)\Omega^{(2)}(x_1, \ldots, x_n) - y_{k_1} > 0 \Leftrightarrow \max(x_1, \ldots, x_{k_1}) > y_{k_1}.$$

The proof for this latest inequality is elementary in the "$\Leftarrow$" direction. The "$\Rightarrow$" implication is equivalent to proving that $y_{k_1} \geqslant \max(x_1, \ldots, x_{k_1}) \Rightarrow \alpha\Omega^{(1)}(x_1, \ldots, x_n) + (1-\alpha)\Omega^{(2)}(x_1, \ldots, x_n) - y_{k_1} \leqslant 0$. This is also straightforward since $y_{k_1} \geqslant \max(x_1, \ldots, x_{k_1})$ implies that $\Omega^{(1)}(x_1, \ldots, x_n) = y_{k_1}$ and therefore $\Omega^{(2)}(x_1, \ldots, x_n) \geqslant \max(x_1, \ldots, x_{k_1})$.

From this inequality it is obvious that $B_1(x_1, \ldots, x_{k_1}) = \max(x_1, \ldots, x_{k_1})$ solves the problem for bidder 1. A similar argument applies for bidders 2 through l. □

This result obviously implies Theorem 1. As mentioned above, for $\alpha = 0$, there exist other asymmetric equilibria:

Claim 7. *For $\alpha = 0$, $B(x_1, s) = x_1$ and $\beta(x_i) = x_i$ for $i \neq 1$ is also an equilibrium.*

Proof. If bidders $2, \ldots, n$ bid β, which is still a best response for them, bidder 1 strictly prefers to win if and only if $x_1 > \max\{x_2, \ldots, x_n\}$, making $B(x_1, s) = x_1$ a best response. □

Note that a similar argument can show that the informed bidder's equilibrium strategy for $\alpha = 0$ can be any bid between $\min\{x_1, s\}$ and $\max\{x_1, s\}$. On the other hand if $\alpha > 0$, $\max\{x_1, s\}$ is a unique best response for bidder 1 and therefore

$B(x_1, s) = \alpha \max\{x_1, s\} + (1-\alpha)x_1$ or $B(x_1, s) = x_1$ are not best responses to $\beta(x_i) = x_i$ for $i \neq 1$, since in the event $\{s > \max(x_2, \ldots, x_n) > \alpha s + (1-\alpha)x_1\}$, bidder 1 loses the auction although he can make strictly positive profits by bidding $\max\{x_1, s\}$. This consideration eliminates the equilibrium just claimed from serious consideration, as no corresponding equilibrium exists when $\alpha > 0$.

Any partition-proof equilibrium must have the property that for $k_1 = 1$, $b_1^*(x_1) = x$, as this is the unique symmetric equilibrium when each bidder observes a single signal [11]. Hence, the equilibrium in Theorem 1 is the unique partition-proof equilibrium for $\alpha > 0$.

Appendix B

Consider an additive model (Eq. (5)), where S_1 and S_2 are i.i.d. $N(0,1)$. Consider the revelation policy "tell bidder 1, $S_1 + \varepsilon_1$ and bidder 2, $S_2 + \varepsilon_2$" where the noise terms $\in_i$ are independent from S_1 and S_2 and also i.i.d. $N(0,1)$. To begin computing the equilibrium bids in a second-price auction under this information revelation mechanism, let

$$\begin{aligned}\tilde{v}(t, s : x_1, x_2) &= E[V \mid X_1 = x_1, S_1 + \varepsilon_1 = t, X_2 = x_2, S_2 + \varepsilon_2 = s] \\ &= x_1 + x_2 + \frac{t}{2} + \frac{s}{2}\end{aligned}$$

based on the simple property that for i.i.d. variables X and Y, $E[X \mid X + Y] = (X+Y)/2$. The obvious symmetric equilibrium bid under these circumstances is

$$b(t, x) = \tilde{v}(t, t : x, x) = 2\left(x + \frac{t}{2}\right).$$

Ex ante, equilibrium revenue under this information structure is going to be

$$2E[\min(X_1 + \bar{S}_1, X_2 + \bar{S}_2)],$$

where $\bar{S}_i = (S_i + \varepsilon_i)/2$. The revenue under "truthful" private revelation is as before

$$2E[\min(X_1 + S_1, X_2 + S_2)].$$

In the case described above (standard normal) assume for simplicity that $X_1, X_2 = 0$ and observe that

$$\frac{S_i + \varepsilon_i}{2} \sim \frac{1}{\sqrt{2}} N(0,1).$$

This means that

$$2E[\min(\bar{S}_1, \bar{S}_2)] = \frac{2}{\sqrt{2}} E[\min(S_1, S_2)] > 2E[\min(S_1, S_2)] < 0,$$

which shows an example in which a noisy private revelation does better than the truthful private revelation. Note that in this case the truthful public announcement can do better than any private revelation scheme.

Appendix C

Note that

$$H(x) = E[X|X \leqslant x] = \frac{\int_0^x sf(s)\,ds}{F(x)} = \frac{xF(x) - \int_0^x F(s)\,ds}{F(x)} = x - \frac{\int_0^x F(t)\,dt}{F(x)}$$

and by a similar argument

$$\tilde{H}(x) = E[X+S|X+S \leqslant x] = x - \frac{\int_0^x F_{X+S}(t)\,dt}{F_{X+S}(x)}.$$

To establish the desired conclusion, consider some preliminary results:

Claim 8. *If F and G are positive continuous functions such that $F(x)/G(x)$ is decreasing in x, then*

$$\frac{\int_0^x F(s)\,ds}{\int_0^x G(s)\,ds} \geqslant \frac{F(x)}{G(x)}$$

and $(\int_0^x F(s)\,ds / \int_0^x G(s)\,ds)$ *is decreasing in x.*

Proof. By Lagrange's Theorem there exists $\xi \in [0, x]$ such that

$$\frac{\int_0^x F(s)\,ds}{\int_0^x G(s)\,ds} = \frac{F(\xi)}{G(\xi)} \geqslant \frac{F(x)}{G(x)}.$$

This property also implies that

$$\frac{\int_0^x F(s)\,ds}{\int_0^x G(s)\,ds}$$

is decreasing in x, since there exists $\eta \in [x, x+y]$

$$\frac{\int_0^x F(s)\,ds}{\int_0^x G(s)\,ds} \geqslant \frac{F(x)}{G(x)} \geqslant \frac{\int_x^{x+y} F(s)\,ds}{\int_x^{x+y} G(s)\,ds} = \frac{F(\eta)}{G(\eta)}$$

and hence

$$\frac{\int_0^{x+y} F(s)\,ds}{\int_0^{x+y} G(s)\,ds} = \frac{\int_0^x F(s)\,ds + \int_x^{x+y} F(s)\,ds}{\int_0^x G(s)ds + \int_x^{x+y} G(s)\,ds} \leqslant \frac{\int_0^x F(s)\,ds}{\int_0^x G(s)\,ds}. \qquad \square$$

Next consider the reversed hazard rate ordering (see [21]) defined as

$$X \leqslant_{rh} Y \Leftrightarrow \frac{F_X(t)}{F_Y(t)} \text{ decreases in } t \text{ over the union of the supports of } X \text{ and } Y.$$

Note that $0 \leqslant_{rh} S$ for any random variable with positive support. We will use the following:

Theorem 9 (Shaked and Shanthikumar [21]). *If the random variables X and Y are such that $X \leqslant_{rh} Y$, and if Z is a random variable independent of X and Y and has a decreasing reversed hazard rate, then $X + Z \leqslant_{rh} Y + Z$.*

As such, under the assumptions of the Claim above, $X \leqslant_{rh} S + X$. This means that $F_X(t)/F_{S+X}(t)$ is decreasing, and furthermore

$$\frac{\int_0^x F_X(t)\,dt}{\int_0^x F_{S+X}(t)\,dt}$$

is decreasing. Taking the derivative of the logarithm of the last expression

$$\frac{d}{dx}\ln\left(\frac{\int_0^x F_X(t)\,dt}{\int_0^x F_{S+X}(t)\,dt}\right) \leqslant 0 \Leftrightarrow \frac{F_X(x)}{\int_0^x F_X(t)\,dt} \leqslant \frac{F_{X+S}(x)}{\int_0^x F_{X+S}(t)\,dt}$$

$$\Leftrightarrow x - \frac{\int_0^x F_{X+S}(t)\,dt}{F_{X+S}(x)} \geqslant x - \frac{\int_0^x F_X(t)\,dt}{F_X(x)}.$$

Combining the above inequalities yields the desired result that $\tilde{H}(x) \geqslant H(x)$.

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