Reinhard Selten (Ed.)

Game Equilibrium Models II

Methods, Morals, and Markets

With Contributions by
D. Abreu, W. Albers, K. Binmore, R. Gardner
W. Güth, R. M. Harstad, H. Kliemt, W. Leininger
A. Okada, E. Ostrom, D. Pearce, L. Phlips
S. H. Schanuel, L. K. Simon, J. Sobel, G. Stephan
E. van Damme, J. W. Walker, F. Weissing, W. R. Zame

With 79 Figures

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona
Budapest

INTERACTION BETWEEN RESOURCE EXTRACTION AND FUTURES MARKETS: A GAME-THEORETIC ANALYSIS[1]

by

Louis Phlips and Ronald M. Harstad

Abstract: This paper is an attempt to model three salient features of futures markets for exhaustible resources: oligopolistic control of the market for the basic commodity, the purely speculative nature of most of futures trading and apparent availability of unlimited funds to the traders.

Interaction between the extraction policy of duopolists and their futures trading is modelled with the help of a two-stage noncooperative game with incomplete information. The second stage is a two-period extraction game, which shows how futures positions taken by the producers affect their planned extraction rates and the resulting spot price of the resource. The first stage is a noncooperative futures game in which equilibrium net positions of and subgame perfect contracts between two producers and a representative speculator are determined.

The purely speculative part of futures positions depends mainly on the (known) structure of inconsistent prior beliefs about the level of uncertain market demand for the basic commodity. The hedging part is related to (known) levels of stocks of the resource available to the individual producers, and to their degree of risk aversion. The conditions under which each player goes net short or net long are worked out in detail.

0. Introduction

This paper is an attempt to model three features which, according to casual observation, characterize futures markets for exhaustible resources such as petroleum, tin or copper, and perhaps a number of other futures markets as well.

On the production side, the markets for the basic commodity are controlled by a few dominant producers: there is oligopolistic competition rather than perfect competition. These oligopolists determine the time path of their extraction rates. Simultaneously they can and do take positions on the futures markets for the commodity they extract. The profitability of these positions depends upon the difference between current futures price and the spot price that will materialize at maturity. This spot price, however, is to some extent under their control, since it depends at least in part upon their extraction policy. As a consequence, extraction

[1] This paper was written while the authors were visiting the Center for Interdisciplinary Research, Bielefeld, W. Germany. We are grateful for the Center's support, and for helpful comments from T. Brianza, E. van Damme, J.-F. Richard, Y. Richelle and R. Selten.

policies of the oligopolists are likely to influence profitability of their futures positions. Conversely, futures positions taken are likely to influence extraction policies and the time path of the spot price.

Secondly, the spectacular development of futures markets suggests that whoever wants to be active on these markets can find necessary funds without difficulty. The organizational set-up is such that only a small fraction of the value of futures transactions has to be available in cash. In addition, futures positions are closed out day by day, including the maturity date, so that actual payments are only a small fraction of the values involved. On top of this, banks apparently readily provide credit to finance these fractions -- until there is a crash, as exemplified by the 1985 tin crisis (see House of Commons (1986) and Anderson and Gilbert (1988)) or, more generally, by the October 1987 stock exchange crisis. This suggests that, for practical purposes, agents active on futures markets (be they producers, traders or speculators) can act (and can be modelled) as if they had no budget constraint.

A third feature which strikes the eye is the purely speculative nature of the majority of transactions on futures markets. Whether this is a recent phenomenon or not is immaterial. The point is that the traditional literature on futures is cast in terms of producers (competitive farmers are favorite examples) who use futures as a hedge against a possibly unfavorable spot price (at harvest time). It seems appropriate to focus analysis instead on the purely speculative part of futures positions, which are not related to producers' initial stocks of the natural resource.

We analyze the situation as a two-stage noncooperative game with inconsistent incomplete information. The production stage is modelled, in the pages that follow, as a Cournot duopoly game. The strategies are extraction rates of a homogeneous exhaustible resource (for which there can be only one equilibrium spot price in each time period). The game is played over the shortest possible number of periods, namely two. The second period is the maturity date of futures contracts made in the first period.

The equilibrium futures prices and the equilibrium futures positions are determined in a game played simultaneously in the first period. This is a three-person game between the two (risk-neutral or risk-averse) producers (call them A and B) and one representative (risk-neutral) speculator (S). All players know the rules of the extraction game played by the two producers. There is uncertainty for all about the level of aggregate demand for the exhaustible resource. The futures game is positive-sum (in terms of expected profit) whenever positions taken are sufficiently small relative to differences in prior beliefs about the level of aggregate demand. These beliefs are common knowledge.

The "futures game" thus leads at each of its "terminations" (sets of agreed-upon contracts) into an extraction game which is a proper subgame. Equilibrium payoffs in on game vary with the futures positions taken, but all these subgames :on structure, and a unique equilibrium extraction path is determined for)ath is compared with a monopoly extractor's behavior when he speculates es market (drawn from Brianza, Phlips and Richard (1987)).

:eed in the standard way, backward, by considering the extraction game hen exploring in Section II the futures game leading to it.

:tion Game

s, A and B, of an exhaustible resource have known initial stocks of $_{0}$. They each face the intertemporal constraint that the initial stock)leted in period 2, or

$$s_{a0} = q_{a1} + q_{a2} \tag{1.a}$$

$$s_{b0} = q_{b1} + q_{b2} , \tag{1.b}$$

: = 1,2) being the extraction rate of producer i in period t. Each exhibit the same instantaneous inverse market demand curve

$$p_t(q_{at} + q_{bt}) = \frac{\alpha}{\beta} - \frac{q_{at} + q_{bt}}{\beta} \qquad t = 1,2 \tag{2}$$

) and $\beta > 0$). In other words, any quantity extracted is immediately sumed: there is no demand for speculative stocks. This admittedly strong ; made to simplify the analysis. (In real life, speculators of course not sell futures, but also buy the commodity in period 1 and carry it over 2 when they expect the spot price p_t to rise. Introducing this vould complicate the analysis unnecessarily.)

futures market, producers A and B have sold futures contracts to the n (possibly negative) amounts f_{as} and f_{bs} at prices p_{as} and p_{bs}. at price p_{ab}, A has sold to B a quantity $f_{ab} = -f_{ba}$ on the ot. We maintain the convention that $f_{ij} > 0$ if i has sold to j with j has sold futures to i (i,j = A,B,S). Thus, $f_{ij} > 0$ indicates a on for i and a long position for j, at least on this contract. (Also, :vention that a doubly subscripted price is a futures price.) Details of iation of these positions and prices are left to Section II. These late to the producers' expectations α_a and α_b about the level of mand, α, which is the only unknown parameter in the extraction game. action paths are determined at the beginning of period 1, before α is

To focus on futures market impact on extraction paths, we simplify by setting the interest rate to zero. Equilibrium extraction paths can then be readily compared to the constant extraction paths $\frac{s_{10}}{2}$ obtained in the absence of a futures market. Producer A's intertemporal profit function is

$$\pi_a = p_2(q_{a2} + q_{b2})(q_{a2} - f_{ab} - f_{as}) + p_1(q_{a1} + q_{b1})q_{a1} + p_{ab}f_{ab} + p_{as}f_{as}, \quad (3.a)$$

which sums extraction sales and net profits of futures contracts. Similarly, producer B has the intertemporal profit function

$$\pi_b = p_2(q_{a2} + q_{b2})(q_{b2} + f_{ab} - f_{bs}) + p_1(q_{a1} + q_{b1})q_{b1} - p_{ab}f_{ab} + p_{bs}f_{bs}, \quad (3.b)$$

where the signs on f_{ab} terms are reversed because $f_{ab} > 0$ represents a short position for A, but a long position for B. Extraction costs are zero or constant and equal between the duopolists.

Payoffs in the extraction game are the certainty equivalent levels of profit:

$$W_a = E_a[\pi_a] - \frac{K_a}{2\beta^2} \operatorname{var}(\alpha_a)(s_{a0} - f_{ab} - f_{as})^2 \quad (4.a)$$

$$W_b = E_b[\pi_b] - \frac{K_b}{2\beta^2} \operatorname{var}(\alpha_b)(s_{b0} + f_{ab} - f_{bs})^2, \quad (4.b)$$

where $K_i \geq 0$ is the degree of constant absolute risk aversion. Equilibrium results from simultaneous maximization of (4.a) and (4.b) subject to (1.a), (1.b).

Notice the implications of depletion constraints (1.a) and (1.b) in this respect. Since there are only two periods, it is always optimal to extract what is left under the ground in period 2, whatever actual extraction rates were in period 1. There is thus no problem of time consistency. Second, no strategic commitments can be made for period 2 and so-called "path strategies" extending over the two-period horizon – in the terminology of Reinganum and Stokey (1985) – are ruled out. Third, backward induction is not necessary to find the subgame perfect equilibrium decision rules: it suffices to solve the (four equation) system of first-order conditions using $q_{a2} = s_{a0} - q_{a1}$ and $q_{b2} = s_{b0} - q_{b1}$. Equilibrium exists since each producer's marginal revenue declines as the extraction of the other producer increases (Novshek (1985)).

Maximization of Lagrangians corresponding to (4.a) and (4.b) with $p_t(\cdot)$ specified as in Equation (2) gives

$$\frac{\alpha - 2q_{a1} - q_{b1}}{\beta} = \frac{\alpha - 2q_{a2} + (f_{ab} + f_{as}) - q_{b2}}{\beta} = \eta_a \tag{5.a}$$

and

$$\frac{\alpha - 2q_{b1} - q_{a1}}{\beta} = \frac{\alpha - 2q_{b2} + (f_{bs} - f_{ab}) - q_{a2}}{\beta} = \eta_b \tag{5.b}$$

where η_a and η_b are multipliers associated with (1.a) and (1.b). The unique equilibrium strategies are[3]

$$q_{a1} = \frac{s_{a0}}{2} + \frac{f_{bs} - f_{ab}}{6} - \frac{f_{ab} + f_{as}}{3} \tag{6.a}$$

$$q_{b1} = \frac{s_{b0}}{2} + \frac{f_{ab} + f_{as}}{6} - \frac{f_{bs} - f_{ab}}{3} \tag{6.b}$$

$$q_{a2} = \frac{s_{a0}}{2} - \frac{f_{bs} - f_{ab}}{6} + \frac{f_{ab} + f_{as}}{3} \tag{6.c}$$

$$q_{b2} = \frac{s_{b0}}{2} - \frac{f_{ab} + f_{as}}{6} + \frac{f_{bs} - f_{ab}}{3} \tag{6.d}$$

which sum to total extraction rates

$$q_1 = \frac{s_{a0} + s_{b0}}{2} - \frac{f_{as} + f_{bs}}{6} \tag{7.a}$$

$$q_2 = \frac{s_{a0} + s_{b0}}{2} + \frac{f_{as} + f_{bs}}{6} \quad . \tag{7.b}$$

According to Eqs. (6.a)-(6.d), taking a net short position $(f_{ab} + f_{as} > 0)$ implies that producer A pumps less in period 1 and more in period 2 than in the absence of a futures market. A long position carries the reverse implication. If A's competitor also has a net short position $(f_{bs} - f_{ab} > 0)$, this counteracts the reduction in A's extraction rate in period 1 and the increase in period 2. As the counteraction is less pronounced, contracts with the speculator impact on the industry's extraction path. Extraction is delayed if the speculator takes a net long position, and vice versa.

The effect on equilibrium prices is clear from the following equations:

$$p_1 = \hat{p} + \theta(f_{as} + f_{bs}) \tag{8.a}$$

$$p_2 = \hat{p} - \theta(f_{as} + f_{bs}) \tag{8.b}$$

[3]Equations (6) assume an interior maximum.

where $\hat{p} = \alpha/\beta-(s_{a0}+s_{b0})/2\beta$ and $\theta = 1/6\beta > 0$. A combined net short position increases p_1 and reduces p_2. In a world of increasing spot prices -- with a positive real interest rate -- this means that the variability of the spot price is reduced (or reversed). A combined net long position has the opposite effect.

At this point, a comparison with the monopoly case is in order. A monopolist with initial stock $(s_{a0}+s_{b0})$ and futures position $(f_{as}+f_{bs})$ extracts (taken from Brianza, Phlips and Richard, 1987):

$$Q_1 = \frac{s_{a0}+s_{b0}}{2} - \frac{f_{as}+f_{bs}}{4}$$

$$Q_2 = \frac{s_{a0}+s_{b0}}{2} + \frac{f_{as}+f_{bs}}{4}$$

and sets prices

$$P_1 = \hat{p} + \frac{3}{2}\,\theta(f_{as}+f_{bs})$$

$$P_2 = \hat{p} - \frac{3}{2}\,\theta(f_{as}+f_{bs}).$$

Notice that in a zero-interest-rate world without a futures market, duopoly achieves the same extraction and price paths as monopoly.

More importantly, with a futures market the impact of producers' futures activity upon *both* total extraction rates and prices is more pronounced in monopoly than in duopoly. In a world of increasing spot prices, *the variability of prices is larger under duopoly than under monopoly when the duopolists take a combined net short position.*

Of course, the duopolists expect to profit from the futures contracts they sign. Some of this profit is passed on to demanders of the resource being extracted. That is, relative to the extraction and price paths without a futures market, Eqs. (7) and (8) show some extraction moved from a (relatively) higher-priced to a (relatively) lower-priced period.

2. The Futures Game

We consider a noncooperative game with 3 players, producers A and B and the speculator S. At the beginning of period 1, a futures market is opened and contracts signed in accordance with rules specified as follows:

Step 1: A "commitment order", which is a permutation of the order {A,B,S}, is chosen randomly, each permutation with equal probability. This commitment order is common knowledge.

imultaneously, each player is allowed to place on the market ("announce") ontract offer and/or one purchase contract offer. Each offer consists of ative but finite (price, quantity) pair. Once announced, the offers are .

e player designated first in the commitment order irrevocably accepts or h contract offered by another player.

e player designated second in the commitment order irrevocably accepts or contract offered by another player (and not yet accepted).

e last player irrevocably accepts or rejects each contract offered by er (and not yet accepted). The futures market closes.

producers then play the extraction subgame.

demand level α is revealed.

ative rule configurations could be analyzed, this structure clarifies between futures activity and production while maintaining simplicity.[3]

offs

producers A and B are given by the certainty equivalent profit .a) and (4.b). The speculator's payoff is

$$\phi_s = (p_2-p_{as})f_{as} + (p_2-p_{bs})f_{bs} \tag{9}$$

> 0 is a futures purchase by S from producer i).

above that expectations about α do not affect the extraction subgame, he combined assumptions that α is the same in both periods and that must be depleted across the two periods. In the futures game, play a central role, which can be clarified by expressing beliefs as about $\hat{\beta}$ rather than α. Let the mean expectations be $\hat{\beta}_a$ and $\hat{\beta}_b$ icers, $\hat{\beta}_s$ for the speculator. Since an outside observer cannot separate (K_i) from confidence of beliefs (var $\hat{\beta}_i$) in producer behavior, we etting $\mathrm{var}(\hat{\beta}_i) = \mathrm{var}(\hat{\beta}) = 1$.

e extraction game to follow is common knowledge, its unique subgame-ibrium strategies (6.a)-(6.d) and (8.a)-(8.b) can be incorporated in the

---

intermediate steps after step 2 and before step 3 could be added, during ts offered could be irrevocably accepted or tentatively rejected. In the consider, there would not be any acceptances "before the deadline" The possibility of improving one's offer (selling at a lower price, or igher price) could also be introduced between steps 2 and 3, and would impact on subgame-perfect equilibria.

payoff functions:

$$W_a = s_{a0}\,\hat{p}_a + V_a \tag{10.a}$$

$$W_b = s_{b0}\,\hat{p}_b + V_b \tag{10.b}$$

$$\phi_s = (\hat{p}_s - p_{as})f_{as} + (\hat{p}_s - p_{bs})f_{bs} - \theta(f_{as} + f_{bs})^2 \tag{10.c}$$

where

$$V_a = (p_{ab} - \hat{p}_a)f_{ab} + (p_{as} - \hat{p}_a)f_{as} + \frac{\theta}{3}(f_{as} + f_{bs})^2 - \frac{K_a}{2}(s_{a0} - f_{ab} - f_{as})^2 \tag{11.a}$$

$$V_b = (\hat{p}_b - p_{ab})f_{ab} + (p_{bs} - \hat{p}_b)f_{bs} + \frac{\theta}{3}(f_{as} + f_{bs})^2 - \frac{K_b}{2}(s_{b0} + f_{ab} - f_{bs})^2 \tag{11.b}$$

(recall that $f_{ij} > 0$ implies i sells futures to j).[4] Because $s_{i0}\hat{p}_i$ is the expected value of the stock in the absence of a futures market, and is unaffected by futures trading, it is useful to focus on V_a, V_b and ϕ_s.

One striking way in which the futures and extraction markets interact shows up in Eqs. (11). Even if producer A chooses to be inactive in the futures market, its existence adds to his profit, as $V_a = (\theta/3)(f_{bs})^2$ when $f_{ab} = f_{as} = 0$. This is because a producer with a zero net short position rationally responds to his competitor's net position by shifting extraction to the period with a higher (discounted) price.

2.2. Beliefs and Attitudes Towards Risk

The payoffs are common knowledge; therefore players' beliefs about $\hat{p}$ are common knowledge. Under rational expectations, this common knowledge would make speculative futures trading impossible -- see the Groucho Marx theorem by Milgrom and Stokey (1982). We therefore abandon rational expectations and assume that the players have different prior beliefs, based on pure differences in *opinion*, in the terminology of Varian (1987). (Note that the differences in beliefs are not due to private information.) No player is inclined to alter his beliefs based upon the knowledge that rivals have different beliefs; each believes he can "outperform the market" because he is smarter than the others. The model thus forms a game of inconsistent incomplete information, as formalized in Selten (1982), Sections B.9-B.13.

[4]Equations (10) and (11) assume the extraction subgame has an interior maximum [illegible] (6) are all positive. Each term involving θ becomes more complex otherwise.

beliefs do not affect equilibrium extraction rates, other than through
sitions, these differences in opinion are compatible with subgame-perfect
1.

sume throughout that the producers are risk-averse. Why this is necessary
1 by examining the sums of the expected profits of two traders. The sums
= a,b, are positive (if the net positions are small enough relative to
in beliefs) and quadratic in f_{is}, so that trading of finite amounts is
etween a producer and the speculator. This is true even under risk
($K_a = K_b = 0$). However, the sum of the payoffs of the two producers is

$$V_b = (\hat{p}_b - \hat{p}_a)f_{ab} + (p_{as} - \hat{p}_a)f_{as} + (p_{bs} - \hat{p}_b)f_{bs} + \frac{2\theta}{3}(f_{as} + f_{bs})^2 \quad (12)$$

$K_b = 0$. This sum is linear in f_{ab}, so that risk-neutral producers would
ited amounts among themselves.

plication of risk aversion is that the futures positions in our model will
be related to the available stocks of the natural resource. Consequently,
market is not purely speculative in the sense of Kreps (1977) and Tirole
equilibrium futures positions will contain a speculative *and* a hedging
This result contrasts with the finding of Brianza, Phlips and Richard
purely speculative trading is possible between a risk-neutral speculator
neutral monopolist with inconsistent prior beliefs.[8]

speculative futures activity by producers does occur in our model, in the
trivializing the extraction subgame via zero resource stocks would not
utures trading by A and B.[9] Because the differences in beliefs
ifferences in opinion, the speculative components of futures contracts
tive sum component of aggregate payoffs (for sufficiently small futures

ures Positions

ubgame-perfect equilibria will be provided in Section 2.4 below. Here, we
net futures positions that will result from equilibrium contracts.

---

culative trading would be possible with risk-neutral duopolists, as
ocks would not appear in Eqs. (11). As mentioned, however, equilibrium
positions would require an artificial constraint on the size of contracts
producers.

al expectations, purely speculative futures trading with a monopolist is
in equilibrium -- see Anderson and Sundaresan (1984). In other words,
tent prior beliefs the monopolist must view the futures market as a
rument.

ade the speculator risk-neutral for simplicity. Note, though, that
risk aversion for S would reduce the size of hedging components
producers' speculative components in net positions.

Suppose equilibrium behavior will reach step 5 of the game (at the end of which the futures market closes), and for concreteness, S will be deciding whether or not to accept a futures contract offered by A, A having rejected any contracts offered by S. There will be some minimum profit level that S requires to be just indifferent between accepting or rejecting A's contract; this level may depend on the (p_{bs}, f_{bs}) contract.

Because A and S agree that their different beliefs and A's hedging options yield potential gains to trade, it will pay A to have announced a contract yielding S enough profit to assure acceptance. We avoid problems that sets of preferred outcomes are open by presuming that a player who otherwise could not trade accepts a contract when he is indifferent.

Thus, the equilibrium contract A should have announced is the (p_{as}, f_{as}) pair which maximizes his profit subject to S receiving his minimum acceptable profit, taking appropriate account of the f_{ab} and f_{bs} contracts. Indeed, once it is clear which contracts will be rejected, the appropriate f_{ab}, f_{as}, f_{bs} levels to announce result from simultaneous solution of the maximization of (V_a+V_b), (ϕ_s+V_a), and (ϕ_s+V_b) with respect to these f_{ij}'s. This system of equations is:

$$(K_a+K_b)f_{ab} + K_a f_{as} - K_b f_{bs} = \hat{p}_b - \hat{p}_a + K_a s_{a0} - K_b s_{b0} \tag{13.a}$$

$$-K_a f_{ab} - (K_a + \tfrac{4\theta}{3})f_{as} - \tfrac{4\theta}{3} f_{bs} = \hat{p}_a - \hat{p}_s - K_a s_{a0} \tag{13.b}$$

$$K_b f_{ab} - \tfrac{4\theta}{3} f_{as} - (K_b + \tfrac{4\theta}{3})f_{bs} = \hat{p}_b - \hat{p}_s - K_b s_{b0} \,. \tag{13.c}$$

Equation (13.c) is the sum of Eqs. (13.a) and (13.b): there is an indeterminacy. However, take two equations, for example (13.a) and (13.c), and rewrite these as

$$f_{ab} + f_{as} = \frac{1}{K_a}(\hat{p}_b-\hat{p}_a) + s_{a0} - \frac{K_b}{K_a} s_{b0} + \frac{K_b}{K_a}(f_{bs}-f_{ab}) \tag{14.a}$$

$$f_{as} + f_{bs} = \frac{3}{4\theta}(\hat{p}_s-\hat{p}_b) + \frac{3K_b}{4\theta} s_{b0} - \frac{3K_b}{4\theta}(f_{bs}-f_{ab}). \tag{14.b}$$

This two-equation system is in terms of the *net* futures positions of the players. Indeed, $f_{ab} + f_{as}$ is A's net position, $f_{bs} - f_{ab}$ is B's net position and $f_{as} + f_{bs}$ is the speculator's net position. Subtraction of (14.a) from (14.b) gives $f_{bs} - f_{ab}$ which can be substituted back, so that the net short positions are uniquely determined as follows:

for A:

$$f_{ab} + f_{as} = \sigma^{-1}[3K_b(\hat{p}_s - \hat{p}_a) + 4\theta(\hat{p}_b - \hat{p}_a) + (\sigma - \sigma_b)s_{a0} - \sigma_b s_{b0}] \qquad (15.a)$$

for B:

$$f_{bs} - f_{ab} = \sigma^{-1}[3K_a(\hat{p}_s - \hat{p}_b) - 4\theta(\hat{p}_b - \hat{p}_a) + (\sigma - \sigma_a)s_{b0} - \sigma_a s_{a0}] \qquad (15.b)$$

for S:

$$-f_{as} - f_{bs} = \sigma^{-1}[-3K_a(\hat{p}_s - \hat{p}_b) - 3K_b(\hat{p}_s - \hat{p}_a) - (\sigma - \sigma_a - \sigma_b)(s_{a0} + s_{b0})] \qquad (15.c)$$

where $\sigma_i = 4\theta K_i$, $\sigma = \sigma_a + \sigma_b + 3K_aK_b$ (a negative amount indicates a net long position). Each net position is seen to be composed of a speculative part and a hedging part. The speculative component consists of the first two terms inside the brackets and is based on differences in beliefs, independent of stocks. The hedging component comprises the remaining terms which relate the futures positions to the available stocks of resources, and are independent of differences in beliefs.[7]

Pricing of speculative ventures is shown to be purely redistributive in a particularly striking way: Eqs. (15) do not contain the futures prices. In other words, contract curves in futures are vertical.

Note that a producer's net short position increases with his own stock and decreases with his competitor's stock of resources.[8] The speculator accommodates the hedging via a less short net position[9] when producers' stocks are increased. This hedging uses the futures market as an insurance or risk-sharing mechanism, and necessarily involves some futures sales to S.

Define the equilibrium net speculative positions to be the first two terms in (15.a), (15.b), (15.c). The signs and magnitudes of net speculative positions directly reflect differences in beliefs. The most optimistic player takes a net speculative long position, and vice versa: the player with the lowest $\hat{p}_i$ will be net speculative short. The player with median beliefs takes the smallest net speculative position. (Here, and throughout, we use "small" as an abbreviation for "small in absolute value", with "large" correspondingly used.) *Increasingly divergent beliefs lead to larger net speculative positions.*

[7]Equations (13)-(15) have assumed an interior maximum in the extraction subgame. The more complex equations that result when a producer finds nonnegative extraction a binding constraint do have his stock appearing in the speculative components.

[8]This suggests that the rival i to a dominant firm j does little hedging. Note, though, that if firm j has a larger stock but is also less risk-averse, this mitigates the effect. (In a many-period model with one depletion constraint, a dominant firm may not be constrained to a large depletion in any short time period, and may have a longer time horizon which could look like a lesser degree of risk aversion.)

[9]If the net position were long, "less short" would mean larger in absolute value.

Should a producer be more risk averse, in equilibrium he takes a smaller net speculative position (i.e., he moves closer to exact hedging of his stock), with both other players accommodating. (In particular, suppose $\hat{p}_a < \hat{p}_b < \hat{p}_s$ and B is net speculative short. Then increasing K_b leads A to a shorter and S to a less short net speculative position.) To complete comparative statics, a steeper demand curve (a higher θ) leads both producers to take less short net positions.[10]

Seemingly small changes in parameters can have notable effects upon net positions, extraction paths, and deviations from constant prices. We illustrate with some examples in the columns of Table 1. Each lists the parameters, followed by net

<u>Table 1</u>

$\hat{p}_a$		14.	14.	14.	14.	14.	14.	14.	14.
$\hat{p}_b$		15.	16.	15.	15.	15.	20.	20.	20.
$\hat{p}_s$		20.	20.	20.	20.	20.	15.	15.	15.
K_a		0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
K_b		0.05	0.05	0.05	0.05	0.05	0.05	0.25	0.12
θ		0.15	0.15	0.75	0.15	0.15	0.75	0.15	0.15
s_{a0}		75.	75.	75.	40.	110.	75.	75.	75.
s_{b0}		75.	75.	75.	110.	40.	75.	75.	75.
net	A	30.56	39.44	14.51	-4.44	65.56	60.85	-8.45	18.
short	B	10.56	-0.56	-5.49	45.56	-24.44	-59.15	34.31	1.25
position	S	-41.11	-38.89	-9.02	-41.11	-41.11	-1.71	-25.86	-19.25
specul.	A	22.22	31.11	12.68	22.22	22.22	59.02	20.	33.
net sht.	B	2.22	-8.89	-7.32	2.22	2.22	-60.98	-20.	-36.25
position	S	-24.44	-22.22	-5.37	-24.44	-24.44	1.95	0.	3.25
q_{a1}		29.07	24.26	31.75	29.07	29.07	7.36	46.03	31.71
q_{b1}		39.07	44.26	41.75	39.07	39.07	67.36	24.66	40.0[illegible]
$p_1 - \hat{p}$		6.17	5.83	6.77	6.17	6.17	1.28	3.88	[illegible]

[10]Increasing speculator risk aversion (as mentioned in footnote 6) has the [illegible] impact as increasing θ.

ositions (net long if negative) of the three players. Below that are the net :ive short positions, followed by each producer's equilibrium extraction rate od 1 (which contrasts with $s_{10}/2$ in the absence of a futures market). The em is the price increase in period 1 relative to constant prices (p_2 is :d by the same amount).

game-Perfect Equilibria

described above has many equilibria. This section will provide the equations rizing subgame-perfect equilibria, and then show how the characterizations a detailed analysis of the commitment order.[11]

subgame-perfect equilibria include equilibrium behavior in the extraction Eqs. (6) are satisfied. For reasons mentioned in Section 2.3 and detailed ne net positions satisfy Eqs. (15).

equilibrium, a player offering a contract naturally prefers it to be accepted er player to going unfulfilled. The player accepting the contract must prefer s to foregoing any trade with the contract offerer. The (weak) preferences oting contracts yield the following equations (where a boldface subscript ; the player accepting the contract):

$$p_{a\mathbf{b}} \gtrless \hat{p}_a - K_a\left(s_{a0} - f_{as} - \frac{f_{ab}}{2}\right) \qquad \text{as } f_{ab} \gtrless 0 \quad (16.a)$$

$$p_{\mathbf{a}b} \lessgtr \hat{p}_b - K_b\left(s_{b0} - f_{bs} + \frac{f_{ab}}{2}\right) \qquad \text{as } f_{ab} \gtrless 0 \quad (16.b)$$

$$p_{a\mathbf{s}} \gtrless \hat{p}_a - K_a\left(s_{a0} - f_{ab} - \frac{f_{as}}{2}\right) - \frac{\theta}{3}\left(f_{as} + 2\, f_{bs}\right) \qquad \text{as } f_{as} \gtrless 0 \quad (16.c)$$

$$p_{\mathbf{a}s} \lessgtr \hat{p}_s - \theta\left(f_{as} + 2\, f_{bs}\right) \qquad \text{as } f_{as} \gtrless 0 \quad (16.d)$$

$$p_{b\mathbf{s}} \gtrless \hat{p}_b - K_b\left(s_{b0} + f_{ab} - \frac{f_{bs}}{2}\right) - \frac{\theta}{3}\left(f_{bs} + 2\, f_{as}\right) \qquad \text{as } f_{bs} \gtrless 0 \quad (16.e)$$

$$p_{\mathbf{b}s} \lessgtr \hat{p}_s - \theta\left(f_{bs} + 2\, f_{as}\right) \qquad \text{as } f_{bs} \gtrless 0. \quad (16.f)$$

---

ernative game-theoretic approach to modelling a futures market would be ve, i.e., view the futures game as a bargaining problem. A variant of the g set (Aumann and Maschler (1964)) could serve the role of resolving the nacy in (13). The cooperative concept needed would be different from the g set or other formalizations we have seen in several ways: it would have to coalition formation and contractual agreement, to allow for variable gains to a producer who is not a member of a coalition as a result of that 's bargaining, and to obtain predictions about agreements in the face of ent incomplete information. It would be preferable if the bargaining model prices or shadow prices without a presumption of a uniform price. (P.t.o.)

The direction of inequalities is simply that a seller prefers a higher, and a buyer a lower price. Equation (16.a), for example, is derived by solving

$$V_a - V_a\Big|_{f_{ab}=0} \geq 0$$

and (16.d) similarly from ϕ_s.

Equations (16) have to do with accepting each contract separately. A player also has the option of foregoing futures activity entirely, by refusing to offer or to sign any contracts. Thus, equilibrium must satisfy

$$V_a \geq \frac{\theta}{3}(f_{bs})^2 \tag{17.a}$$

$$V_b \geq \frac{\theta}{3}(f_{as})^2 \tag{17.b}$$

$$\phi_s \geq 0 \tag{17.c}$$

where the first two terms recognize that a contract between S and one producer benefits the other producer even if he is inactive in futures.

We discard two types of equilibrium as implausible because they are not subgame perfect. First, there are equilibria where (6.a)-(6.d) are not satisfied for all (f_{ab}, f_{as}, f_{bs}). In essence, these equilibria involve threats by one or both producers to punish deviations from the equilibrium path via altering their extraction policy to harm both themselves and the player being threatened. For example, there may be an equilibrium where S takes a net long position of 30 units. If (6.a)-(6.d) were satisfied everywhere, S would prefer a net long position of 45 units. However, A has threatened to extract 0 in period 1 and s_{a0} in period 2 (rather than profit maximizing) if S takes any net long position of more than 30 units. Notice that A does not have to carry out this threat in equilibrium, as S takes the 30-unit position.

Such threats are not credible and such equilibria are not sensible. The extraction game occurs after the futures market is closed, and now A and B derive higher certainty equivalent payoffs from satisfying (6.a)-(6.d) than from any unilateral deviation. They have no way to have committed themselves to any alternative extraction plan while the futures market was still open.

Development of such a cooperative solution concept is both beyond the scope and somewhat counter to the purposes of this paper. Bargaining seems necessarily to lead to outcomes which are not at all reflective of the structure of futures markets operations, and might naturally lead to joint agreements about contracts and extraction paths. Such joint agreements would yield little insight about the interaction between futures and cash markets.

there are equilibria which fail to be subgame-perfect within the itself. Equilibrium in itself does not take the commitment order into ver another player ends up rejecting your contract offers, your best accept his terms. But if you are deciding whether to accept or reject ing step 3, subgame perfectness expects you to consider what his ior will be in the event that you reject his terms. In this event, the ilibrium behavior that he rejects your terms during step 4 or 5 is le discard the equilibrium where you accept his terms in step 3, in ubgame-perfect equilibrium where he accepts your terms after you have jected his.

ft with infinitely many subgame-perfect equilibria, characterized as atisfies (6.a) - (6.d), and, of course, (17.a) - (17.c). A player only act if no alternative means of securing any gains from trade with the s. This property can be discerned by any contract offerer, who unces contract terms which maximize his payoff subject to the contract le, i.e., subject to one of Eqs. (16). As discussed, such contracts satisfy Eqs. (15), so *all subgame-perfect equilibria yield positions*. Equilibrium contracts also yield the accepting player no off, so the relevant equation in (16) is satisfied with equality.[12]

e the sequence of subgame-perfect actions which yield (15), (16) and lesignate the player active during step 3, j the player active during k the player active during step 5, and work backwards. Since at the the futures market closes, player k can do no better than to accept fered by any player with whom k has not traded.

p 4, player j rejects any contract offered by k, as j prefers s own contract to k's terms. In the event that step 3 saw i reject offer, j can do no better than to accept the contract (clearly m) offered by i.

p 3, i rejects a contract offered by j or k if i prefers that rms be accepted. If i has a short position on the contract with j tion with k, or vice versa, then i rejects all contracts. Suppose, i has a short position on both contracts, or long on both. Then he pportunity to announce his preferred terms for one of these contracts (Recall only a player taking a short position on one contract and a

---

although contracts are offered to the market and not identified as to the acceptor, in equilibrium the acceptor is known at announcement.

1 Eq. (16) can be thought of as the limit of a sequence of contract e increasingly profitable to the offerer and all of which the player s to accept. An alternative approach would be to introduce a smallest nd require all prices to be integer multiples of this unit. No nges would result, save that acceptance need never be the result of otation and calculation would be complicated considerably.

long position on his other can announce two contracts which could possibly be accepted.) He rationally would have chosen to announce his major contract (ij if $|(p_{ij}-p_{ij})f_{ij}| > |(p_{ik}-p_{ik})f_{ik}|$). Now, during step 3, he can do no better than to accept the terms dictated him on his minor contract.

During step 2, contract offers that will later be accepted are announced, in accordance with (15) and (16) as described. Formally, it does not matter whether contracts rejected in equilibrium satisfy (15) (though they must be acceptable to the offerer), or whether these contracts are even announced.

Averaging across commitment orders, a player who takes a short position in one contract and a long position in his other, in all commitment orders, obtains the gains from exchange in each of the two contracts 5 times out of 6 (if B is this player, for example, only in order ASB does S dictate terms to B, and only in order SAB does A dictate to B). The gains from exchange between a player who only takes short positions and one who only takes long positions, across commitment orders, accrue to each 3 times out of 6.

3. Obtaining a Unique Set of Contracts

Differences in beliefs about the uncertain level of resource demand, producers' stocks of resources and levels of risk aversion, and the slope of the resource demand curve suffice to determine uniquely the traders' net positions, the extraction rates, and the divergence from constant prices. Yet these parameters leave open an infinity of equilibrium options for the prices and quantities written on futures contracts, and for the distribution of payoff levels across players.

None of the subgame-perfect equilibria strike us as particularly implausible, so reducing to a unique prediction is not an equilibrium refinement task in the usual sense.[13] A unique prediction can be attained by a revision of the rules described in Section 2, without altering any of the analysis of Section 3.

Specifically, we remove the word "simultaneously" from the description of contract offer announcement in step 2, and substitute a rule that the player chosen (at random) to be first in the commitment order also is allowed to be first in announcing any contract offers.[14]

This rule alteration yields a unique subgame-perfect equilibrium. Being first to commit, player i is also first to announce. Let Y_i be the function obtained [illegible]

[13]Without extensive exploration, it appears to us that most refinements discussed [illegible] the literature (see van Damme (1987) for a clear exposition) leave us with [illegible] infinite set. One approach could be to follow the equilibrium selection methods [illegible] Harsanyi and Selten (1988); to do so would require alteration of the game to fi[illegible] strategy sets. Our approach here is simpler and more transparent, but may [illegible] robust to changes in the structure of the game.

[14]Other methods of breaking announcement simultaneity may also yield [illegible] prediction in a less straightforward manner.

vant equations in (15) and (16) are substituted for prices and all but one position in i's payoff function (V_a, V_b or ϕ_s). Y_i is a quadratic mapping any one variable, f_{im}, into the payoff i obtains if he the contract offer pair (f_{im}, p_{im}) derived from an equality in (16), and ucceeding announcements are in accordance with (15) and (16), and contract e behavior is as described in Section 2.4. If Eqs. (17) are satisfied, the he subgame following announcement (f_{im}, p_{im}) which leads to payoff $Y_i(f_{im})$ s a subgame-perfect path. (If i takes short positions (or long positions) ontracts, set f_{im} to be his major contract.)

layers can discern that i will reject any contract offers by m in step g m to accept (f_{im}, p_{im}), that is, to accept i's terms. Accordingly, deviations from the prices and quantities derived from (15) and (16) rease payoffs.

the f_{im} which maximizes Y_i subject to (17) is i's best choice for announcement.[15] With divergent beliefs, $0 < \theta < \infty$ and at least one e player, contracts with both other players represent potential gains to player i, so he cannot prefer an f_{im} failing to satisfy (17). In this announcement choice is unconstrained.

uding Perspective

ceding computations, the true level of aggregate demand and the correct f the spot market price do not appear. Each contract has its own futures price. Consequently, there is no sense in which "the" futures be a predictor, unbiased or biased, of the spot price to materialize at ith inconsistent beliefs, rather than private information, each player has utable opinion about how the spot price will behave in period 2; he does the futures price to provide any information to update this opinion. As market" does not "aggregate" information in the sense these words are lied in futures market models. Whether a strength or a weakness, this is a of our approach.

ifferences of a game-theoretic approach to futures market analysis are engths from an industrial organization point of view: game theory allows ake account of the strategic interdependence characteristic of oligopoly raction between cash and futures markets, and (b) to introduce explicitly l arrangements that characterize operation of futures markets and trace mpact on market outcomes. While some of our simplifications were more to

---

aightforward, though tedious, to show that equations (17) bound f_{im}. happens to achieve a global minimum exactly at the midpoint of upper and will the best f_{im} fail to be unique.

clarify than to describe, the empirical relevance of basic noncooperative elements of the structure should be clear.

Indeed, a formalization of the problem as a two-stage game and resulting characterization of subgame-perfect equilibrium outcomes appears a natural way to model the interaction between oligopolistic control of spot markets for natural resources and manipulation of corresponding futures markets. We find that the existence of futures markets can be profit-enhancing to oligopolists in general, and that profitable speculation (as well as hedging) transforms the temporal evolution of extraction rates and the spot price. In particular, the model shows how a futures position taken by a competitor can be a source of profit for an individual producer, whether he is active in the futures market or not. Equilibrium outcomes include the striking and perhaps descriptive possibilities that a risk-averse producer can take a net long futures position (his net speculative long position being larger than his hedging position) and even a net long hedging position (when a competitor has a larger stock to deplete).

The futures market is modelled as an oral double auction ("open cry") in rough description of operating rules of such markets as the London Metal Exchange. While the particular commitment order rule we use is more transparent than some complex but realistic alternatives, it does capture the serious phenomenon that minor details of timing can have major impacts on profit distributions. A reassuring feature of equilibrium outcomes in this model is that only profit distributions depend upon commitment order; many equilibria exist, but equilibrium net futures positions and extraction paths are uniquely determined by stocks of resources, risk aversion, demand responsiveness and differences in beliefs.

The oral double auction avoids the untenable assumption of a single uniform price in a futures market. Thus, our simple story reflects more naturally the multiplicity of prices (often among the same traders) during one trading day on, say, the London Metal Exchange. This is one of a number of realistic divergences from the standard capital asset pricing model in vogue in the futures markets literature. The futures market in subgame-perfect equilibrium is efficient, in the sense that net positions taken maximize the sum of payoffs, but no efficient markets assumption is employed. Finally, the usual rational expectations assumption (which precludes examining interactions between producers' net speculative positions and interdependent extraction policies) is abandoned in favor of inconsistent prior beliefs. We are pleased to discover that the differences in beliefs play an essential role in determining the sign and size of net speculative positions.

The presumption that the underlying commodity is a depletable resource played a role for which other technological relations or constraints could be substituted. In many markets trading in futures or other financial instruments, we see underlying oligopolistic industrial structures, a high volume of essentially speculative ıd participants with apparent access to unlimited credit. A game-theoretic › analyzing these markets should be fruitful as well.

R.W., ed. (1984). The Industrial Organization of Futures Markets. ton, Mass: D.C. Health and Company.

R.W. and C.L. Gilbert (1988). Commodity Agreements and Commodity Markets - s from Tin. The Economic Journal 98: 1-15.

R.W. and M. Sundaresan (1984). Futures Markets and Monopoly, In: R.W. on (ed.): The Industrial Organization of Futures Markets. Lexington, D.C. Health and Company, pp: 75-106.

J. and M. Maschler (1964). The Bargaining Set for Cooperative Games. In: sher, L.S. Shapley and A.W. Tucker (eds.): Advances in Game Theory. Annals them. Studies No 52. Princeton, N.J.: Princeton University Press, 3-476.

., L. Philips and J.-F. Richard (1987). Futures Markets, Speculation and ly". Revised version of CORE Discussion Paper 8725, Louvain-la-Neuve.

J.C. and R. Selten (1988). A General Theory of Equilibrium Selection in Cambridge, Mass.: MIT Press.

ommons (1986). The Tin Crisis. Second Report from the Trade and Industry tee, Session 1985-86. London: HMSO.

(1977). A Note on 'Fulfilled Expectations' Equilibria. J. Econom. Theory 43.

.R. and N.L. Stokey (1982). Information, Trade and Common Knowledge. om. Theory 26: 17-27.

(1985). On the Existence of Cournot Equilibrium. Rev. Econom. Studies 98.

J.F. and N.L. Stokey (1985). Oligopoly Extraction of a Common Property Resource: the Importance of the Period of Commitment in Dynamic Games. Econom. Rev. 26: 161-173.

(1982). Einführung in die Theorie der Spiele mit unvollständiger tion. Information in der Wirtschaft, Schriften des Vereins für olitik, Gesellschaft für Wirtschafts- und Sozialwissenschaften, N.F. 5: 81-147, Berlin: Duncker & Humblot.

(1982). On the Possibility of Speculation Under Rational Expectations. trica 24: 74-81.

E.E.C. (1987). Stability and Perfection of Nash Equilibria. Berlin: r-Verlag.

R. (1989). Differences in Opinion in Financial Markets. In: C. Stone Financial Risk: Theory, Evidence and Implications. Amsterdam: Kluwer c Publishers.