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Game Equilibrium Models II
Methods, Morals, and Markets

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INTERACTION BETWEEN RESOURCE EXTRACTION AND FUTURES MARKETS:
A GAME-THEORETIC ANALYSIS

by

Louis Philips and Ronald M. Harstad

Abstract: This paper is an attempt to model three salient features of futures markets for exhaustible resources: oligopolistic control of the market for the basic commodity, the purely speculative nature of most of futures trading and apparent availability of unlimited funds to the traders.

Interaction between the extraction policy of duopolists and their futures trading is modelled with the help of a two-stage noncooperative game with incomplete information. The second stage is a two-period extraction game, which shows how futures positions taken by the producers affect their planned extraction rates and the resulting spot price of the resource. The first stage is a noncooperative futures game in which equilibrium net positions of and subgame perfect contracts between two producers and a representative speculator are determined.

The purely speculative part of futures positions depends mainly on the (known) structure of inconsistent prior beliefs about the level of uncertain market demand for the basic commodity. The hedging part is related to (known) levels of stocks of the resource available to the individual producers, and to their degree of risk aversion. The conditions under which each player goes net short or net long are worked out in detail.

0. Introduction

This paper is an attempt to model three features which, according to casual observation, characterize futures markets for exhaustible resources such as petroleum, tin or copper, and perhaps a number of other futures markets as well.

On the production side, the markets for the basic commodity are controlled by a few dominant producers: there is oligopolistic competition rather than perfect competition. These oligopolists determine the time path of their extraction rates. Simultaneously they can and do take positions on the futures markets for the commodity they extract. The profitability of these positions depends upon the difference between current futures price and the spot price that will materialize at maturity. This spot price, however, is to some extent under their control, since it depends at least in part upon their extraction policy. As a consequence, extraction...
policies of the oligopolists are likely to influence profitability of their futures positions. Conversely, futures positions taken are likely to influence extraction policies and the time path of the spot price.

Secondly, the spectacular development of futures markets suggests that whoever wants to be active on these markets can find necessary funds without difficulty. The organizational set-up is such that only a small fraction of the value of futures transactions has to be available in cash. In addition, futures positions are closed out day by day, including the maturity date, so that actual payments are only a small fraction of the values involved. On top of this, banks apparently readily provide credit to finance these fractions -- until there is a crash, as exemplified by the 1985 tin crisis (see House of Commons (1986) and Anderson and Gilbert (1988)) or, more generally, by the October 1987 stock exchange crisis. This suggests that, for practical purposes, agents active on futures markets (be they producers, traders or speculators) can act (and can be modelled) as if they had no budget constraint.

A third feature which strikes the eye is the purely speculative nature of the majority of transactions on futures markets. Whether this is a recent phenomenon or not is immaterial. The point is that the traditional literature on futures is cast in terms of producers (competitive farmers are favorite examples) who use futures as a hedge against a possibly unfavorable spot price (at harvest time). It seems appropriate to focus analysis instead on the purely speculative part of futures positions, which are not related to producers' initial stocks of the natural resource.

We analyze the situation as a two-stage noncooperative game with inconsistent incomplete information. The production stage is modelled, in the pages that follow, as a Cournot duopoly game. The strategies are extraction rates of a homogeneous exhaustible resource (for which there can be only one equilibrium spot price in each time period). The game is played over the shortest possible number of periods, namely two. The second period is the maturity date of futures contracts made in the first period.

The equilibrium futures prices and the equilibrium futures positions are determined in a game played simultaneously in the first period. This is a three-person game between the two (risk-neutral or risk-averse) producers (call them A and B) and one representative (risk-neutral) speculator (S). All players know the rules of the extraction game played by the two producers. There is uncertainty about all about the level of aggregate demand for the exhaustible resource. The futures market is a positive-sum (in terms of expected profit): whenever positions taken are sufficiently small relative to differences in prior beliefs about the level of aggregate demand. These beliefs are common knowledge.

The "futures game" thus leads at each of its terminations (sets of agreed contracts) into an extraction game which is a proper subgame. Equilibrium payoffs...
on game vary with the futures positions taken, but all these subgames on structure, and a unique equilibrium extraction path is determined for path is compared with a monopoly extractor’s behavior when he speculates as market (drawn from Brianza, Philips and Richard (1987)).

seed in the standard way, backward, by considering the extraction game then exploring in Section II the futures game leading to it.

Action Game

$s_i$, $A$ and $B$, of an exhaustible resource have known initial stocks of $s_0$. They each face the intertemporal constraint that the initial stock depleted in period 2, or

$$s_{a0} = q_{a1} + q_{a2}$$

(1.a)

$$s_{b0} = q_{b1} + q_{b2},$$

(1.b)

$:$ = 1,2) being the extraction rate of producer i in period t. Each exhibit the same instantaneous inverse market demand curve

$$p_t(q_{at} + q_{bt}) = \frac{a}{\beta} \cdot \frac{q_{at} + q_{bt}}{\beta}$$

(2)

$\beta > 0$. In other words, any quantity extracted is immediately consumed; there is no demand for speculative stocks. This admittedly strong assumption made to simplify the analysis. (In real life, speculators of course not sell futures, but also buy the commodity in period 1 and carry it over to 2 when they expect the spot price $p_t$ to rise. Introducing this would complicate the analysis unnecessarily.)

Futures market, producers $A$ and $B$ have sold futures contracts to the $n$ (possibly negative) amounts $f_{as}$ and $f_{bs}$ at prices $p_{as}$ and $p_{bs}$: at price $p_{ab}$. $A$ has sold to $B$ a quantity $f_{ab} = -f_{ba}$ on the st. We maintain the convention that $f_{ij} > 0$ if $i$ has sold to $j$ with $j$ has sold futures to $i$ (1, j = A, B, S). Thus, $f_{ij} > 0$ indicates a short position for $i$ and a long position for $j$, at least on this contract. (Also, convention that a doubly subscripted price is a futures price.) Details of formation of these positions and prices are left to Section II. These relate to the producers’ expectations $o_a$ and $o_b$ about the level of demand, $o$, which is the only unknown parameter in the extraction game. Action paths are determined at the beginning of period 1, before $o$ is
To focus on futures market impact on extraction paths, we simplify by setting the interest rate to zero. Equilibrium extraction paths can then be readily compared to the constant extraction paths \( \frac{s_0}{2} \) obtained in the absence of a futures market. Producer A's intertemporal profit function is

\[
x_a = p_a(q_{a2} + q_{b2})(q_{a2} - f_{ab} - f_{as}) + p_1(q_{a1} + q_{b1})q_{a1} + p_{ab} f_{ab} + p_{as} f_{as},
\]

which sums extraction sales and net profits of futures contracts. Similarly, producer B has the intertemporal profit function

\[
x_b = p_a(q_{a2} + q_{b2})(q_{b2} + f_{ab} - f_{bs}) + p_1(q_{a1} + q_{b1})q_{b1} - p_{ab} f_{ab} + p_{bs} f_{bs},
\]

where the signs on \( f_{ab} \) terms are reversed because \( f_{ab} > 0 \) represents a short position for A, but a long position for B. Extraction costs are zero or constant and equal between the duopolists.

Payoffs in the extraction game are the certainty equivalent levels of profit:

\[
v_a = E_a[x_a] - \frac{K_a}{2\sigma^2} \text{var}(q_a)(s_{a0} - f_{ab} - f_{as})^2
\]

\[
v_b = E_b[x_b] - \frac{K_b}{2\sigma^2} \text{var}(q_b)(s_{b0} + f_{ab} - f_{bs})^2,
\]

where \( K \geq 0 \) is the degree of constant absolute risk aversion. Equilibrium results from simultaneous maximization of (4.a) and (4.b) subject to (1.a), (1.b).

Notice the implications of depletion constraints (1.a) and (1.b) in this respect. Since there are only two periods, it is always optimal to extract what is left under the ground in period 2, whatever actual extraction rates were in period 1. There is thus no problem of time consistency. Second, no strategic commitments can be made for period 2 and so-called "path strategies" extending over the two-period horizon — in the terminology of Reinganum and Stokay (1985) — are ruled out. Third, backward induction is not necessary to find the subgame perfect equilibrium decision rules: it suffices to solve the (four equation) system of first-order conditions using \( q_{a2} = s_{a0} - q_{a1} \) and \( q_{b2} = s_{b0} - q_{b1} \). Equilibrium exists since each producer's marginal revenue declines as the extraction rate of the other producer increases (Novshek (1985)).

Maximization of Lagrangians corresponding to (4.a) and (4.b) with specified as in Equation (2) gives
\[
\frac{a-2q_{a1}}{\beta} - \frac{a-2q_{a2}}{\beta} = \eta_a
\]
(5.a)

\[
\frac{a-2q_{b1}}{\beta} - \frac{a-2q_{b2}}{\beta} = \eta_b
\]
(5.b)

where \(\eta_a\) and \(\eta_b\) are multipliers associated with (1.a) and (1.b). The unique equilibrium strategies are

\[
q_{a1} = \frac{s_{a0}}{2} + \frac{f_{bs} - f_{as} + f_{ab} + f_{as}}{3}
\]
(6.a)

\[
q_{b1} = \frac{s_{b0}}{2} - \frac{f_{bs} - f_{as} + f_{ab} + f_{as}}{3}
\]
(6.b)

\[
q_{a2} = \frac{s_{a0}}{2} - \frac{f_{bs} - f_{ab} + f_{ab} + f_{as}}{3}
\]
(6.c)

\[
q_{b2} = \frac{s_{b0}}{2} - \frac{f_{ab} + f_{as} + f_{bs} - f_{ab}}{3}
\]
(6.d)

which sum to total extraction rates

\[
q_1 = \frac{s_{a0} + s_{b0}}{2} - \frac{f_{as} + f_{bs}}{6}
\]
(7.a)

\[
q_2 = \frac{s_{a0} + s_{b0}}{2} - \frac{f_{as} + f_{bs}}{6}
\]
(7.b)

According to Eqs. (6.a)-(6.d), taking a net short position \((f_{ab} + f_{as} > 0)\) implies that producer A pumps less in period 1 and more in period 2 than in the absence of a futures market. A long position carries the reverse implication. If A's competitor also has a net short position \((f_{bs} - f_{ab} > 0)\), this counteracts the reduction in A's extraction rate in period 1 and the increase in period 2. As the counteraction is less pronounced, contracts with the speculator impact on the industry's extraction path. Extraction is delayed if the speculator takes a net long position, and vice versa.

The effect on equilibrium prices is clear from the following equations:

\[
p_1 = \bar{p} + \theta(f_{as} + f_{bs})
\]
(8.a)

\[
p_2 = \bar{p} - \theta(f_{as} + f_{bs})
\]
(8.b)

Equations (8) assume an interior maximum.
where $\beta = a/\beta(s_{0}+s_{b})/2\theta$ and $\theta = 1/6\theta > 0$. A combined net short position increases $p_1$ and reduces $p_2$. In a world of increasing spot prices -- with a positive real interest rate -- this means that the variability of the spot price is reduced (or reversed). A combined net long position has the opposite effect.

At this point, a comparison with the monopoly case is in order. A monopolist with initial stock $(s_{a0}+s_{b0})$ and futures position $(f_{as}+f_{bs})$ extracts (taken from Brianza, Philips and Richard, 1987):

$$Q_1 = \frac{s_{a0}+s_{b0}}{2} - \frac{f_{as}+f_{bs}}{4}$$

$$Q_2 = \frac{s_{a0}+s_{b0}}{2} + \frac{f_{as}+f_{bs}}{4}$$

and sets prices

$$P_1 = \beta + \frac{3}{2} \theta (f_{as}+f_{bs})$$

$$P_2 = \beta - \frac{3}{2} \theta (f_{as}+f_{bs})$$.

Notice that in a zero-interest-rate world without a futures market, duopoly achieves the same extraction and price paths as monopoly.

More importantly, with a futures market the impact of producers' futures activity upon both total extraction rates and prices is more pronounced in monopoly than in duopoly. In a world of increasing spot prices, the variability of prices is larger under duopoly than under monopoly when the duopolists take a combined net short position.

Of course, the duopolists expect to profit from the futures contracts they sign. Some of this profit is passed on to demanders of the resource being extracted. That is, relative to the extraction and price paths without a futures market, Eqs. (7) and (8) show some extraction moved from a (relatively) higher-priced to a (relatively) lower-priced period.

2. The Futures Game

We consider a noncooperative game with 3 players, producers A and B and the speculator S. At the beginning of period 1, a futures market is opened and contracts signed in accordance with rules specified as follows:

Step 1: A "commitment order", which is a permutation of the order $\{A,B,S\}$, is chosen randomly, each permutation with equal probability. This commitment is common knowledge.

Step 2: If one sale occurs, the market is closed.

Step 3: The market rejects each contract.

Step 4: The market rejects each contract.

Step 5: The market rejects each contract.

Step 6: The market rejects each contract.

Step 7: The market rejects each contract.

While altering interaction:

2.1 The Payoffs

For functions (4), (recall $f_{as}$ and $f_{bs}$).

We saw that given stocks expectations expectations for the producer aversion to risk aversion imply by substituting the function $\tilde{f}_{as}$ and $\tilde{f}_{bs}$ since the market will accept one contract.

Notice that the contract will be accepted if the market.

Steps 3-5. Ending at a high price, there is no any...
Simultaneously, each player is allowed to place on the market ("announce") a contract offer and/or one purchase contract offer. Each offer consists of a positive but finite (price, quantity) pair. Once announced, the offers are irrevocable.

The player designated first in the commitment order irrevocably accepts or rejects the contract offered by another player.

The player designated second in the commitment order irrevocably accepts or rejects the contract offered by another player (and not yet accepted).

The last player irrevocably accepts or rejects each contract offered by the first two players (and not yet accepted). The futures market closes.

The demand level \( D \) is revealed.

The relative rule configurations could be analyzed, this structure clarifies the demand level between futures activity and production while maintaining simplicity.

\[ \text{offs} \]

Producers \( A \) and \( B \) are given by the certainty equivalent profit (a) and (4.b). The speculator's payoff is

\[ \phi_s = (p_2 - p_{as}) f_{as} + (p_2 - p_{bs}) f_{bs} \]  

(9)

\( > 0 \) is a futures purchase by \( S \) from producer \( i \).

Above that expectations about \( \alpha \) do not affect the extraction subgame, the combined assumptions that \( \alpha \) is the same in both periods and that \( \beta \) must be depleted across the two periods. In the futures game, play a central role, which can be clarified by expressing beliefs as beliefs about \( \beta \) rather than \( \alpha \). Let the mean expectations be \( \bar{\beta}_a \) and \( \bar{\beta}_b \) for producers, \( \bar{\beta}_s \) for the speculator. Since an outside observer cannot separate the \( (K_i) \) from confidence of beliefs (var \( \beta_i \)) in producer behavior, we set \( \text{var}(\beta_a) = \text{var}(\beta_b) = 1 \).

The extraction game to follow is common knowledge, its unique subgame-perfect equilibrium strategies (6.a)-(6.d) and (8.a)-(8.b) can be incorporated in the analysis.  

Intermediate steps after step 2 and before step 3 could be added, during which offers could be irrevocably accepted or tentatively rejected. In the latter case, there would not be any acceptances "before the deadline." The possibility of improving one's offer (selling at a lower price, or higher price) could also be introduced between steps 2 and 3, and would impact on subgame-perfect equilibria.
payoff functions:

\[ W_a = s_{a0} \hat{\rho}_a + V_a \]  \hspace{1cm} (10.a)

\[ W_b = s_{b0} \hat{\rho}_b + V_b \]  \hspace{1cm} (10.b)

\[ \phi_s = (\hat{\rho}_s - p_{as})f_{as} + (\hat{\rho}_s - p_{bs})f_{bs} - 8(f_{as} + f_{bs})^2 \] \hspace{1cm} (10.c)

where

\[ V_a = (p_{ab} - \hat{\rho}_a)f_{ab} + (p_{as} - \hat{\rho}_a)f_{as} + \frac{\theta}{2}(f_{as} + f_{bs})^2 - \frac{K_a}{2}(s_{a0} - f_{ab} - f_{as})^2 \] \hspace{1cm} (11.a)

\[ V_b = (p_{ab} - \hat{\rho}_b)f_{ab} + (p_{bs} - \hat{\rho}_b)f_{bs} + \frac{\theta}{2}(f_{as} + f_{bs})^2 - \frac{K_b}{2}(s_{b0} + f_{ab} - f_{bs})^2 \] \hspace{1cm} (11.b)

(recall that \( f_{ij} > 0 \) implies \( i \) sells futures to \( j \)). Because \( s_{a0} \hat{\rho}_a \) is the expected value of the stock in the absence of a futures market, and is unaffected by futures trading, it is useful to focus on \( V_a \), \( V_b \) and \( \phi_s \).

One striking way in which the futures and extraction markets interact shows up in Eqs. (11). Even if producer \( A \) chooses to be inactive in the futures market, its existence adds to his profit, as \( V_a = (\theta/3)(f_{bs})^2 \) when \( f_{ab} = f_{as} = 0 \). This is because a producer with a zero net short position rationally responds to his competitor's net position by shifting extraction to the period with a higher (discounted) price.

2.2. Beliefs and Attitudes Towards Risk

The payoffs are common knowledge; therefore players' beliefs about \( \hat{\rho} \) are common knowledge. Under rational expectations, this common knowledge would make speculative futures trading impossible -- see the Groucho Marx theorem by Milgrom and Stokey (1982). We therefore abandon rational expectations and assume that the players have different prior beliefs, based on pure differences in opinion, in the terminology of Varian (1987). (Note that the differences in beliefs are not due to private information.) No player is inclined to alter his beliefs based on the knowledge that rivals have different beliefs; each believes he can "outperform" the market because he is smarter than the others. The model thus forms a game of incomplete information, as formalized in Selten (1982), Sections B.9-B.13.

\[ \phi_s = \ldots \]

Equations (10) and (11) assume the extraction subgame has an interior maximum (6) are all positive. Each term involving \( \theta \) becomes more complex otherwise.

Since futures possess

We can see \( V_a + \phi_s \) when differences possible be neutrality

\[ V_a + \phi_s \]

when \( K_a = K \) trade unlimited

The impact turn out to our futures (1982) the component. T (1987) that a risk-averse

Purely sense that eliminate future represent different form a position.

2.3 Net Futures Details of such focus on the

purely speculative stock resource stock with finite price between the producers rationals it possible to consistence arising instants or have maintained requiring relative to price.
beliefs do not affect equilibrium extraction rates, other than through positions, these differences in opinion are compatible with subgame-perfect N.

sume throughout that the producers are risk-averse. Why this is necessary by examining the sums of the expected profits of two traders. The sums \( i = a, b \) are positive (if the net positions are small enough relative to (in beliefs) and quadratic in \( f_{15} \), so that trading of finite amounts is between a producer and the speculator. This is true even under risk (\( K_a = K_b = 0 \)). However, the sum of the payoffs of the two producers is

\[
V_b = (\delta_b - \delta_a) f_{ab} + (p_{as} - \delta_a) f_{as} + (p_{bs} - \delta_b) f_{bs} + \frac{20}{3} (f_{as} + f_{bs})^2
\]

(12)

\( C_{b0} = 0 \). This sum is linear in \( f_{ab} \), so that risk-neutral producers would bid amounts among themselves.

lication of risk aversion is that the futures positions in our model will be related to the available stocks of the natural resource. Consequently, market is not purely speculative in the sense of Kreps (1977) and Tirole equilibrium futures positions will contain a speculative and a hedging This result contrasts with the finding of Brianza, Philips and Richard purely speculative trading is possible between a risk-neutral speculator neutral monopolist with inconsistent prior beliefs.a speculative futures activity by producers does occur in our model, in the trivializing the extraction subgame via zero resource stocks would not futures trading by A and B. Because the differences in beliefs differences in opinion, the speculative components of futures contracts give sum component of aggregate payoffs (for sufficiently small futures futures Positions

subgame-perfect equilibria will be provided in Section 2.4 below. Here, we net futures positions that will result from equilibrium contracts.

ulative trading would be possible with risk-neutral duopolists, as scks would not appear in Eqs. (11). As mentioned, however, equilibrium positions would require an artificial constraint on the size of contracts producers.

al expectations, purely speculative futures trading with a monopolist is in equilibrium -- see Anderson and Sundaresan (1984). In other words, agents' prior beliefs the monopolist must view the futures market as a element.

de the speculator risk-neutral for simplicity. Note, though, that risk aversion for \( S \) would reduce the size of hedging components producers' speculative components in net positions.
Suppose equilibrium behavior will reach step 5 of the game (at the end of which the futures market closes), and for concreteness, S will be deciding whether or not to accept a futures contract offered by A, A having rejected any contracts offered by S. There will be some minimum profit level that S requires to be just indifferent between accepting or rejecting A’s contract; this level may depend on the \( (p_{bs}, f_{bs}) \) contract.

Because A and S agree that their different beliefs and A’s hedging options yield potential gains to trade, it will pay A to have announced a contract yielding S enough profit to assure acceptance. We avoid problems that sets of preferred outcomes are open by presuming that a player who otherwise could not trade accepts a contract when he is indifferent.

Thus, the equilibrium contract A should have announced is the \( (f_{as}, f_{as}) \) pair which maximizes his profit subject to S receiving his minimum acceptable profit, taking appropriate account of the \( f_{ab} \) and \( f_{bs} \) contracts. Indeed, once it is clear which contracts will be rejected, the appropriate \( f_{ab}, f_{as}, f_{bs} \) levels to announce result from simultaneous solution of the maximization of \( (V_{a} + V_{b}) \), \( (\phi_{s} + V_{a}) \), and \( (\phi_{s} + V_{b}) \) with respect to these \( f_{ij} \)’s. This system of equations is:

\[
(K_{a} + K_{b}) f_{ab} + K_{a} f_{as} - K_{b} f_{bs} = \hat{\beta}_{b} - \hat{\beta}_{a} + K_{a} s_{a} - K_{b} s_{b} \quad (13.1)
\]

\[-K_{a} f_{ab} - (K_{a} + \frac{48}{3}) f_{as} - \frac{48}{3} f_{bs} = \hat{\beta}_{a} - \hat{\beta}_{s} - K_{a} s_{a} \quad (13.2)\]

\[K_{b} f_{ab} - \frac{48}{3} f_{as} - (K_{b} + \frac{48}{3}) f_{bs} = \hat{\beta}_{b} - \hat{\beta}_{s} - K_{b} s_{b} \quad (13.3)\]

Equation (13.3) is the sum of Eqs. (13.1) and (13.2): there is an indeterminacy. However, take two equations, for example (13.1) and (13.3), and rewrite these as

\[f_{ab} + f_{as} - \frac{1}{K_{a}} (\hat{\beta}_{b} - \hat{\beta}_{a}) + s_{a} - \frac{K_{b}}{K_{a}} s_{b} + \frac{K_{b}}{K_{a}} (f_{bs} - f_{ab}) \quad (14.1)\]

\[f_{as} + f_{bs} - \frac{3}{48} (\hat{\beta}_{b} - \hat{\beta}_{a}) + \frac{3K_{b}}{48} s_{b} - \frac{3K_{b}}{48} (f_{bs} - f_{ab}) \quad (14.2)\]

This two-equation system is in terms of the net futures positions of the players. Indeed, \( f_{ab} + f_{as} \) is A’s net position, \( f_{bs} - f_{ab} \) is B’s net position, and \( f_{as} + f_{bs} \) is the speculator’s net position. Subtraction of (14.1) from (14.2) gives \( f_{bs} - f_{ab} \), which can be substituted back, so that the net short positions are uniquely determined as follows:
for A:

\[ f_{ab} + f_{as} = \sigma^{-1} [3K_0(\hat{\beta}_s - \hat{\beta}_a) + 48(\hat{\beta}_a - \hat{\beta}_p) + (\sigma - \sigma_a) s_{a0} - \sigma_b s_{b0}] \]  \hspace{1cm} (15.a)

for B:

\[ f_{bs} - f_{ab} = \sigma^{-1} [3K_0(\hat{\beta}_s - \hat{\beta}_p) - 48(\hat{\beta}_b - \hat{\beta}_a) + (\sigma - \sigma_a) s_{b0} - \sigma_a s_{a0}] \]  \hspace{1cm} (15.b)

for S:

\[ -f_{as} - f_{bs} = \sigma^{-1} [-3K_0(\hat{\beta}_s - \hat{\beta}_b) - 3K_0(\hat{\beta}_p - \hat{\beta}_a) - (\sigma - \sigma_a) s_{a0} + \sigma_b s_{b0}] \]  \hspace{1cm} (15.c)

where \( \sigma_i = 48K_0 \), \( \sigma = \sigma_a + \sigma_b + 3K_0 \), and \( \sigma_a = 3K_0 \) (a negative amount indicates a net long position). Each net position is seen to be composed of a speculative part and a hedging part. The speculative component consists of the first two terms inside the brackets and is based on differences in beliefs, independent of stocks. The hedging component comprises the remaining terms which relate the futures positions to the available stocks of resources, and are independent of differences in beliefs.\footnote{Equations (13)-(15) have assumed an interior maximum in the extraction subgame. The more complex equations that result when a producer finds nonnegative extraction a binding constraint do have his stock appearing in the speculative components. This suggests that the rival \( i \) to a dominant firm \( j \) does little hedging. Note, though, that if firm \( j \) has a larger stock but is also less risk-averse, this mitigates the effect. (In a many-period model with one depletion constraint, a dominant firm may not be constrained to a large depletion in any short time period, and may have a longer time horizon which could look like a lesser degree of risk version.)}

Pricing of speculative ventures is shown to be purely redistributive in a particularly striking way: Eqs. (15) do not contain the futures prices. In other words, contract curves in futures are vertical.

Note that a producer’s net short position increases with his own stock and decreases with his competitor’s stock of resources.\footnote{If the net position were long, “less short” would mean larger in absolute value.} The speculator accommodates the hedging via a less short net position\footnote{This suggests that the rival \( i \) to a dominant firm \( j \) does little hedging. Note, though, that if firm \( j \) has a larger stock but is also less risk-averse, this mitigates the effect. (In a many-period model with one depletion constraint, a dominant firm may not be constrained to a large depletion in any short time period, and may have a longer time horizon which could look like a lesser degree of risk version.)} when producers’ stocks are increased. This hedging uses the futures market as an insurance or risk-sharing mechanism, and necessarily involves some futures sales to S.

Define the equilibrium net speculative positions to be the first two terms in (15.a), (15.b), (15.c). The signs and magnitudes of net speculative positions directly reflect differences in beliefs. The most optimistic player takes a net speculative long position, and vice versa: the player with the lowest \( \hat{\beta}_i \) will be net speculative short. The player with median beliefs takes the smallest net speculative position. (Here, and throughout, we use “small” as an abbreviation for “small in absolute value”, with “large” correspondingly used.) Increasingly divergent beliefs lead to larger net speculative positions.

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\( f_{ab} \) is the expected value of the payoff from holding a long position in both A and B, \( f_{as} \) is the expected value of the payoff from holding a long position in A and a short position in S, and so on. The expected value is taken over the probability distribution of beliefs, which is assumed to be uniform over the interval \( [\hat{\beta}_a, \hat{\beta}_p] \). The precise form of the expected value depends on the specific payoff function, which is assumed to be quadratic in the beliefs. The expected value is then the sum of the payoffs weighted by the probability of each belief, which is assumed to be uniform over the interval \( [\hat{\beta}_a, \hat{\beta}_p] \).
Should a producer be more risk averse, in equilibrium he takes a smaller net speculative position (i.e., he moves closer to exact hedging of his stock), with both other players accommodating. (In particular, suppose $\beta_a < \beta_b < \beta_s$ and $B$ is net speculative short. Then increasing $K_b$ leads $A$ to a shorter and $S$ to a less short net speculative position.) To complete comparative statics, a steeper demand curve (a higher $\theta$) leads both producers to take less short net positions.10

Seemingly small changes in parameters can have notable effects upon net positions, extraction paths, and deviations from constant prices. We illustrate with some examples in the columns of Table 1. Each lists the parameters, followed by net

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>$\beta_b$</td>
<td>15.</td>
<td>16.</td>
<td>15.</td>
<td>15.</td>
<td>15.</td>
<td>20.</td>
<td>20.</td>
<td>20.</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>20.</td>
<td>20.</td>
<td>20.</td>
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<td>20.</td>
<td>15.</td>
<td>15.</td>
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<tr>
<td>$K_a$</td>
<td>0.05</td>
<td>0.05</td>
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</tr>
<tr>
<td>$K_b$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.15</td>
<td>0.15</td>
<td>0.75</td>
<td>0.15</td>
<td>0.15</td>
<td>0.75</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$s_{a0}$</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
<td>40.</td>
<td>110.</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
</tr>
<tr>
<td>$s_{b0}$</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
<td>75.</td>
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<td>75.</td>
</tr>
</tbody>
</table>

| net A | 30.56 | 39.44 | 14.51 | -4.44 | 65.56 | 60.85 | -8.45 | 18. |
| net B | 10.56 | -0.56 | -5.49 | 45.56 | -24.44 | -59.15 | 34.31 | 1.75 |
| position $S$ | -41.11 | -38.89 | -9.02 | -41.11 | -41.11 | -1.71 | -25.86 | -19.25 |

specul. A | 22.22 | 31.11 | 12.68 | 22.22 | 22.22 | 59.02 | 20. | 33. |
| net shunt. B | 2.22 | -8.89 | -7.32 | 2.22 | 2.22 | -60.98 | -20. | -36.25 |

10Increasing speculative risk aversion (as mentioned in footnote 6) has an impact as increasing $\theta$.short pc speculat in peric
positions (net long if negative) of the three players. Below that are the net negative short positions, followed by each producer’s equilibrium extraction rate in period 1 (which contrasts with $t_{10}/2$ in the absence of a futures market). The net position is the price increase in period 1 relative to constant prices ($p_2$ is divided by the same amount).

**game-Perfect Equilibria**

described above has many equilibria. This section will provide the equations governing subgame-perfect equilibria, and then show how the characterizations provide a detailed analysis of the commitment order. 14

Subgame-perfect equilibria include equilibrium behavior in the extraction Eqs. (6) are satisfied. For reasons mentioned in Section 2.3 and detailed net positions satisfy Eqs. (15).

equilibrium, a player offering a contract naturally prefers it to be accepted or player to going unfulfilled. The player accepting the contract must prefer to foregoing any trade with the contract offerer. The (weak) preferences of contracts yield the following equations (where a boldface subscript is the player accepting the contract):

\[
P_{ab} \leq \beta_a - K_a (s_{a0} - f_{as} - \frac{f_{ab}}{2}) \quad \text{as } f_{ab} \geq 0 \tag{16.a}
\]

\[
P_{ab} \geq \beta_b - K_b (s_{b0} - f_{bs} + \frac{f_{ab}}{2}) \quad \text{as } f_{ab} \geq 0 \tag{16.b}
\]

\[
P_{as} \leq \beta_a - K_a (s_{a0} - f_{ab} - \frac{f_{as}}{2}) - \frac{1}{3} (f_{as} + 2 f_{bs}) \quad \text{as } f_{as} \leq 0 \tag{16.c}
\]

\[
P_{as} \leq \beta_s - \theta (f_{as} + 2 f_{bs}) \quad \text{as } f_{as} \leq 0 \tag{16.d}
\]

\[
P_{bs} \leq \beta_b - K_b (s_{b0} + f_{ab} - \frac{f_{bs}}{2}) - \frac{1}{3} (f_{bs} + 2 f_{as}) \quad \text{as } f_{bs} \leq 0 \tag{16.e}
\]

\[
P_{bs} \leq \beta_s - \theta (f_{bs} + 2 f_{as}) \quad \text{as } f_{bs} \leq 0 \tag{16.f}
\]

An alternative game-theoretic approach to modelling a futures market would be, i.e., view the futures game as a bargaining problem. A variant of the game (Aumann and Maschler (1964)) could serve the role of resolving the uncertainty in (13). The cooperative concept needed would be different from the game or other formalizations we have seen in several ways: it would have to coalition formation and contractual agreement, to allow for variable gains to a producer who is not a member of a coalition as a result of that producer’s bargaining, and to obtain predictions about agreements in the face of incomplete information. It would be preferable if the bargaining model prices or shadow prices without a presumption of a uniform price. (P.t.o.)
The direction of inequalities is simply that a seller prefers a higher, and a buyer a lower price. Equation (16.a), for example, is derived by solving

\[
V_a - V_a \geq 0
\]

and (16.d) similarly from \( \phi_s \).

Equations (16) have to do with accepting each contract separately. A player also has the option of foregoing futures activity entirely, by refusing to offer or to sign any contracts. Thus, equilibrium must satisfy

\[
V_a \geq \frac{2}{3} (f_{bs})^2 \quad (17.a)
\]

\[
V_b \geq \frac{2}{3} (f_{as})^2 \quad (17.b)
\]

\[
\phi_s \geq 0 \quad (17.c)
\]

where the first two terms recognize that a contract between \( S \) and one producer benefits the other producer even if he is inactive in futures.

Moreover, futures game is not account: whenever response is to his terms during rational behavior postulated equilibrium not credible. In favor of the strategy irreversibly rejected.

We are left to follow. Each \( s \) accepts a counter offerer remainder rationally announce being acceptable payoff-maximal: the same net \( p \) increase in payoffs.

To describe (17), let \( f \) de step 4, and \( a \) of step 5, the contract offer.

During step 4 the terms of his \( J \)’s contract is intended for him.

During step 5 a contract term and a long position however, that is already had the opportunity step 2.

Notice that, a

Development of such a cooperative solution concept is both beyond the scope somewhat counter to the purposes of this paper. Bargaining seems necessarily to outcomes which are not at all reflective of the structure of futures operations, and might naturally lead to joint agreements about contracts the extraction paths. Such joint agreements would yield little insight about the interaction between futures and cash markets.
there are equilibria which fail to be subgame-perfect within the itself. Equilibrium in itself does not take the commitment order into over another player ends up rejecting your contract offers, your best accept his terms. But if you are deciding whether to accept or reject ing step 3, subgame perfectness expects you to consider what his tor will be in the event that you reject his terms. In this event, theilibrium behavior that he rejects your terms during step 4 or 5 is de discard the equilibrium where you accept his terms in step 3, in subgame-perfect equilibrium where he accepts your terms after you have rejected his.

left with infinitely many subgame-perfect equilibria, characterized as satisfies (6.a) - (6.d), and, of course, (17.a) - (17.c). A player only act if no alternative means of securing any gains from trade with the s. This property can be discerned by any contract offerer, who unces contract terms which maximize his payoff subject to the contract able, i.e., subject to one of Eqs. (16). As discussed, such contracts satisfy Eqs. (15), so all subgame-perfect equilibria yield positions. Equilibrium contracts also yield the accepting player no off, so the relevant equation in (16) is satisfied with equality.12 be the sequence of subgame-perfect actions which yield (15), (16) and designate the player active during step 3, j the player active during k the player active during step 5, and work backwards. Since at the the futures market closes, player k can do no better than to accept fered by any player with whom k has not traded.

up 4, player j rejects any contract offered by k, as j prefers own contract to k's terms. In the event that step 3 saw i reject offer, j can do no better than to accept the contract (clearly m) offered by i.

up 3, i rejects a contract offered by j or k if i prefers that terms be accepted. If i has a short position on the contract with j nation with k, or vice versa, then i rejects all contracts. Suppose, i has a short position on both contracts, or long on both. Then he opportunity to announce his preferred terms for one of these contracts (Recall only a player taking a short position on one contract and a although contracts are offered to the market and not identified as to the acceptor, in equilibrium the acceptor is known at announcement.

In Eq. (16) can be thought of as the limit of a sequence of contract increasingly profitable to the offerer and all of which the player is to accept. An alternative approach would be to introduce a smallest and require all prices to be integer multiples of this unit. Ho changes would result, save that acceptance need never be the result of notation and calculation would be complicated considerably.
long position on his other can announce two contracts which could possibly be accepted.) He rationally would have chosen to announce his major contract (if $|p_{ij} - p_{ij}^j| > |p_{ik} - p_{ik}^k|$. Now, during step 3, he can do no better than to accept the terms dictated him on his minor contract.

During step 2, contract offers that will later be accepted are announced, in accordance with (15) and (16) as described. Formally, it does not matter whether contracts rejected in equilibrium satisfy (15) (though they must be acceptable to the offerer), or whether these contracts are even announced.

Averaging across commitment orders, a player who takes a short position in one contract and a long position in his other, in all commitment orders, obtains the gains from exchange in each of the two contracts 5 times out of 6 (if $B$ is this player, for example, only in order ASB does $S$ dictate terms to $B$, and only in order SAB does $A$ dictate to $B$). The gains from exchange between a player who only takes short positions and one who only takes long positions, across commitment orders, accrue to each 3 times out of 6.

3. Obtaining a Unique Set of Contracts

Differences in beliefs about the uncertain level of resource demand, producers’ stocks of resources and levels of risk aversion, and the slope of the resource demand curve suffice to determine uniquely the traders’ net positions, the extraction rates, and the divergence from constant prices. Yet these parameters leave open an infinity of equilibrium options for the prices and quantities written on futures contracts, and for the distribution of payoff levels across players.

None of the subgame-perfect equilibria strike us as particularly implausible, so reducing to a unique prediction is not an equilibrium refinement task in the usual sense. A unique prediction can be attained by a revision of the rules described in Section 2, without altering any of the analysis of Section 3.

Specifically, we remove the word "simultaneously" from the description of contract offer announcement in step 2, and substitute a rule that the player chosen (at random) to be first in the commitment order also is allowed to be first in announcing any contract offers. 14

This rule alteration yields a unique subgame-perfect equilibrium. Being first to commit, player $i$ is also first to announce. Let $Y_i$ be the function obtained.

---

14 Without extensive exploration, it appears to us that most refinements discussed in the literature (see van Damme (1987) for a clear exposition) leave us with infinite set. One approach could be to follow the equilibrium selection method of Harsanyi and Selten (1968); to do so would require alteration of the game to strategy sets. Our approach here is simpler and more transparent, but may be less robust to changes in the structure of the game.

15 Other methods of breaking announcement simultaneity may also yield a prediction in a less straightforward manner.
vant equations in (15) and (16) are substituted for prices and all but one position in \( i \)'s payoff function \( (V_a, V_b, \text{or} \phi) \). \( Y_i \) is a quadratic mapping any one variable, \( f_{im} \), into the payoff \( i \) obtains if he the contract offer pair \((f_{im}, P_{im})\) derived from an equality in (16), and succeeding announcements are in accordance with (15) and (16), and contract behavior is as described in Section 2.4. If Eqs. (17) are satisfied, the \( \frac{d}{dP_{im}}Y_i(f_{im}) \) is a subgame-perfect path. (If \( i \) takes short positions (or long positions) contracts, set \( f_{im} \) to be his major contract.)

Players can discern that \( i \) will reject any contract offers by \( m \) in step 4 to accept \((f_{im}, P_{im})\), that is, to accept \( i \)'s terms. Accordingly, deviations from the prices and quantities derived from (15) and (16) increase payoffs.

The \( f_{im} \) which maximizes \( Y_i \) subject to (17) is \( i \)'s best choice for announcement. With divergent beliefs, \( 0 < b < \infty \) and at least one player contracts with both other players represent potential gains to player \( i \), so he cannot prefer an \( f_{im} \) failing to satisfy (17). In this announcement choice is unconstrained.

Using Perspective

Proceeding computations, the true level of aggregate demand and the correct if the spot market price do not appear. Each contract has its own futures price. Consequently, there is no sense in which "the" futures are a predictor, unbiased or biased, of the spot price to materialize at with inconsistent beliefs, rather than private information, each player has an opinion about how the spot price will behave in period 2; he does for the futures price to provide any information to update this opinion. As a market does not "aggregate" information in the sense these words are in futures market models. Whether a strength or a weakness, this is a of our approach.

Differences of a game-theoretic approach to futures market analysis are strengths from an industrial organization point of view: game theory allows at account of the strategic interdependence characteristic of oligopoly reaction between cash and futures markets, and (b) to introduce explicitly all arrangements that characterize operation of futures markets and trace impact on market outcomes. While some of our simplifications were more to

Sightforward, though tedious, to show that equations (17) bound \( f_{im} \) happens to achieve a global minimum exactly at the midpoint of upper and will the best \( f_{im} \) fail to be unique.
clarify than to describe, the empirical relevance of basic noncooperative elements of the structure should be clear.

Indeed, a formalization of the problem as a two-stage game and resulting characterization of subgame-perfect equilibrium outcomes appears a natural way to model the interaction between oligopolistic control of spot markets for natural resources and manipulation of corresponding futures markets. We find that the existence of futures markets can be profit-enhancing to oligopolists in general, and that profitable speculation (as well as hedging) transforms the temporal evolution of extraction rates and the spot price. In particular, the model shows how a futures position taken by a competitor can be a source of profit for an individual producer, whether he is active in the futures market or not. Equilibrium outcomes include the striking and perhaps descriptive possibilities that a risk-averse producer can take a net long futures position (his net speculative long position being larger than his hedging position) and even a net long hedging position (when a competitor has a larger stock to deplete).

The Futures market is modelled as an oral double auction ("open cry") in rough description of operating rules of such markets as the London Metal Exchange. While the particular commitment order rule we use is more transparent than some complex but realistic alternatives, it does capture the serious phenomenon that minor details of timing can have major impacts on profit distributions. A reassuring feature of equilibrium outcomes in this model is that only profit distributions depend upon commitment order; many equilibria exist, but equilibrium net futures positions and extraction paths are uniquely determined by stocks of resources, risk aversion, demand responsiveness and differences in beliefs.

The oral double auction avoids the untenable assumption of a single uniform price in a futures market. Thus, our simple story reflects more naturally the multiplicity of prices (often among the same traders) during one trading day on, say, the London Metal Exchange. This is one of a number of realistic divergences from the standard capital asset pricing model in vogue in the futures markets literature. The futures market in subgame-perfect equilibrium is efficient, in the sense that net positions taken maximize the sum of payoffs, but no efficient markets assumption is employed. Finally, the usual rational expectations assumption (which precludes examining interactions between producers' net speculative positions and interdependent extraction policies) is abandoned in favor of inconsistent price beliefs. We are pleased to discover that the differences in beliefs play an essential role in determining the sign and size of net speculative positions.

The presumption that the underlying commodity is a depletable resource plays a role for which other technological relations or constraints could be substituted in many markets trading in futures or other financial instruments, we see underlying oligopolistic industrial structures, a high volume of essentially specialized trading, an approach to

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