Modeling Competitive Bidding: A Critical Essay

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In analyzing bidding, modeling matters. This paper is a critical analysis of the models available to aid competitive bidding decision making—bidding strategy and auction design—in real transactions. After an introductory overview, this paper describes the contexts in which auctions arise, reviews the “mainstream” theory of single, isolated auctions and discusses the important work involved in enrichment of this theory. In doing so, it indicates results that have been obtained and the sort of changes in analytical approach that are needed to tackle other critical enrichments. The paper summarizes briefly what is known about the direct use of models by bidders and auction designers. A general theme of this paper is that enriched models are needed to bring bidding theory closer to direct applicability in decision making.

(Games–Group Decisions; Bidding–Auctions; Modeling; Economics; Decision Analysis)

1. Introduction

Bidding is important. The volume of economic transactions conducted by competitive bidding gives importance both to the study of auctions as part of basic research in economics and management science, and to the evaluation of the assistance bidding practitioners can get from the advances made in auction theory. Both concerns gain further importance from the role of competitive bidding as the archetypal formalization of competitive exchange. Those interested in the general topic of decision making in the face of both competition and uncertainty have found auctions and bidding a fruitful area of study. Auctions have the added structure imposed by formal rules of exchange and price determination, and auctions yield extra data (e.g., on unsuccessful bids). Hence, formal results for general economic models have been preceded by qualitatively similar results developed for the special case of auctions.

Our focus in this essay is on competitive bidding decision making—bidding strategy and auction design—in real transactions. This has not been the precise focus of the mainstream theoretical literature on competitive bidding—particularly the game-theoretic literature of the 1980s. Indeed, we find gaps between bidding theory and bidding decision making which seriously limit the direct usefulness of much theory to those who decide how much to bid and those who design auctions. Our central themes are that modeling matters, that context matters, and that elegance and powerful theorems are often insufficient to obtain practical advice. Enrichments to models implemented as steps toward realism can critically alter important inferences.

1.1. Bidding Theory in Economic Context

Economically important auctions are not isolated. Just as the "general equilibrium" tradition in economic theory has highlighted interrelations across markets in an economy, we emphasize that auctions take place in a larger context. Single, isolated auctions are easier to analyze. Models of isolated auctions may serve as useful building blocks, but the isolation can obscure important impacts. Behavior in auctions will tend to be altered by the context in which the auctions arise. Furthermore, the outcome of competitive bidding will typically affect economic relationships external to the market in which the transaction occurs. Indeed, performance measures
such as efficiency (throughout, we use “efficiency” as shorthand for “allocative efficiency”) need to take into account not only auction outcomes, but also the impacts that cascade across markets external to the auction. The literature tends to proceed via incremental contributions to analyses of single, isolated auctions, often presented as if most important auctions were isolated. We believe that a new emphasis on context is starting to reinvigorate bidding research.

1.2. The Bidding Theory / Bidding Phenomena Gap

1.2.1. Overview. Sophisticated bidders in many auction markets profess to having little use for bidding theory. Design issues and objectives, as well as behavioral assumptions, tend to segregate most theoretical contributions in the literature from usefulness to practitioners. For example, a behavioral assumption that underlying probability distributions generating bidders’ information are commonly known is ubiquitous in the literature and often untenable in practice. It is common for bidding theory to analyze situations in which the behavior of a bidder is assumed to be a known (or calculable) monotonic function of his private information, so that anyone observing his bid can infer his information. Maintaining the privacy of such information is typically of much importance to bidders (in large part because the auction does not take place in isolation). Were the assumption correct that rivals could infer completely private information, bidders might well prefer an information-concealing mixed strategy that would lead to reduced expected profitability in a given auction.

For another example, commercial auctioneers have described as naive the common assumption that the auction form is chosen so as to maximize expected revenue. They strive for a balance, seeking high enough prices to keep sellers consigning assets to them for sale, yet low enough prices to keep bidders voluntarily attending in sufficient numbers. Clearly, this is a matter of context: the auctioneer is concerned with the long run health and growth of his business, not with maximizing his commission revenue from a single auction. Fairness and the appearance of fairness are important to commercial auctioneers as well as to public-sector bid-takers.

Finally, bidding theory, following a game-theoretic tradition, tends to take the formalism seriously, and in doing so may lose insights from less rigorous analyses. In practice, it is sometimes useful to think of auctions as more like negotiations conducted according to procedures that are relatively formal by contrast with other negotiations. Parties may seek to change, bend or break rules to their own advantage, and are likely to be successful in such efforts whenever sufficient aspects of mutual advantage are arguable. A few bidding papers have looked at collusion among bidders, but have tended to maintain the same degree of formalism while introducing a topic that is in practice a considerable reduction in formalism. A common practice undressed in modeling is for a high bidder who did not meet a secret reserve price to then enter into negotiations with the seller. This sometimes occurs at the instigation of the auctioneer, whose reduced commission may be the first step in bridging the gap between willingness to pay and to accept.

1.2.2. Result Reversals with Model Enrichment. The phenomena physicists attempt to predict never arise in truly frictionless settings; nonetheless, frictionless models are useful to the extent that their answers approximate observations in low-friction environments. While all models necessarily abstract from some aspects of reality, auction models have shown a striking tendency for the answers to change as enrichments to their realism are introduced. This tendency should discourage attempts to derive general answers in abstract models.

Attempts to enrich mainstream models with a view toward relevance to practice are still a small part of the bidding theory literature, but they suggest that result reversals may be common. Here are three examples, each of which is discussed in more detail below.

First, a canonical question is whether a bidder rationally responds to more competition by bidding more aggressively or less aggressively. In both decision-theoretic and game-theoretic models, as the model was enriched, the answer changed from unambiguously more aggressively to quite likely less aggressively. As a second example, part of the initial appeal of the second-price, sealed-bid auction procedure coined by Vickrey (1961) was that it assured an allocatively efficient outcome under minimal behavioral assumptions; in broader contexts, even stronger assumptions yield less assurance.
As a final introductory example, in many auction models a bid-taker is better off with a greater number of bidders competing. Suppose, however, the model is enriched to have bidders participate only when a rational calculation shows the prospect of winning is worth the cost of participating. Then, in a wide variety of circumstances, the bid-taker prefers policies yielding less bidder participation.

1.3. This Paper
This introduction concludes with a list of other sources on bidding theory and a brief introduction to some terms used to describe auction types. There are then four remaining parts to this paper before a brief conclusion. The first of these tries to describe the contexts in which auctions arise and some of the impacts contexts have upon behavior in auctions and the evaluation of auction outcomes. The next part surveys the mainstream theory of single, isolated auctions. The third part discusses the work involved in enrichment of this theory, indicating results that have been obtained, and the sort of changes in analytical approach that are needed to tackle other critical enrichments. The fourth part is a brief summary of what is known about the direct use of models by bidders and auction designers. A general theme of this paper is that while enriched models may feature less elegant and less encompassing results, they are needed to bring bidding theory closer to direct relevance to decision making. This paper should serve as both a caution flag to those interested in applying the theory and as a welcome mat to those interested in extending it in fruitful directions.

1.4. Sources for Initial Immersions
There is a large, highly dispersed literature on competitive bidding. However, there are a few concentrated sources of wide interest worth mentioning. These include Cassady’s book (1967) describing the wide usage of auctions and variety of oral auction practices, a comprehensive but early and unannotated bibliography by Stark and Rothkopf (1979) and three survey papers—an early, but wide-ranging one by Engelbrecht-Wiggans (1980) and two more recent ones by Milgrom (1985) and McAfee and McMillan (1987). The latter is restricted to game-theoretic models of single isolated auctions. In addition, there are two edited volumes of papers—Amihud (1976) and Engelbrecht-Wiggans, Shubik and Stark (1983), a “Symposium” of four papers in an issue of the Journal of Economic Perspectives (Vol. 3, No. 3, Summer 1989)—a book by Smith (1990) giving a sociological interpretation of oral auctions, Wilson’s (1987) short, meaty entry in The New Palgrave, and handbook chapters by Wilson (1993) and by Rothkopf (forthcoming 1994).

1.5. Types of Auctions
There are many different forms of auctions and several useful ways of classifying these variants. A “standard” auction means one in which the winner is the highest bidder among potential buyers, or the lowest bidder among potential sellers. The distinction between contexts in which bidders are competing to buy and to sell is relatively unimportant: there is an almost perfect correspondence in results. (A caution with respect to this correspondence is the potential role of zero as an implicit limit on acceptable bids. The implications of a failure to transact due to failure of bidders to meet a reserve price may also be distinct.) In what follows, we will normally not comment further on the difference between bidding to buy and bidding to sell.

Most bidding theory papers discuss a single isolated auction of a single indivisible asset. The most common exception is papers discussing an isolated sale, usually simultaneous but perhaps sequential, of a fixed number of identical assets to bidders each of whom attach no value to a second purchase; these are typically called “multunit” auctions. Parallels to single-asset auctions exist, but are incomplete.

Auctions can be either “open” or “closed.” In open auctions, prices are publicly announced and bidders can indicate their willingness to transact at the particular prices. In closed auctions, bidders submit offers simultaneously, and these offers are then evaluated by the bid-taker.

Open auctions can be “progressive” as in the familiar “English auction” in which the auctioneer solicits successively better offers until no bidder is willing make a better bid. Alternatively, they can be “Dutch auctions” as used in Dutch flower markets sales, where the auctioneer (or now an electronic device) announces successively lower prices until a bidder bids and thereby wins the right to the lot for sale.
By far the most common closed auction form is standard sealed bidding. However, there is an alternate form that is used only occasionally. It was proposed by Vickrey (1961) in a partially successful attempt to devise a closed procedure strategically equivalent to the English auction. In such "Vickrey auctions" of single items, the winning bidder is the one who submitted the best bid, but the price is set at the level of the second-best bid. Thus, it is called a "second-price auction" in contrast to standard sealed bidding and Dutch auctions which are called "first-price auctions" since the winner's price is the best price offered. English auctions are considered second-price auctions, because the transaction price is at the level of (or marginally better than) the best price acceptable to any losing bidder. Multiunit analogues are "discriminatory" auctions (each winner pays the price he bid) and "highest-rejected-bid" auctions.

Auctions also blend into request-for-proposal processes. Some such processes are essentially auctions. In others the rules for evaluating proposals are not well understood by the bidders, and the proposals may be the first step in a relatively unstructured negotiation process.

2. Contexts for Auctions: Dynamic, Complex, Political

2.1. Where Is Bidding Used?
Bidding is used widely: in financial markets ranging from tin futures to fine art; in harvest sales to distributors of fish, flowers, leather, tobacco, and many others; in financial settlements of bankruptcies, estates, repossessions, condominium conversions and the like; in public-sector and some private-sector procurement and subcontracting; in the allocation of unique assets such as book manuscripts, antiques, free-agent athletes, etc.; in sales of rights, e.g., for mineral exploration or extraction. In addition, sequential negotiations where an agent negotiates a tentative agreement with one potential transacting partner, and then returns calls to all other potential transacting partners still interested in competing, to see if they can better the tentative terms, may be modeled as auctions.

An auction of a unique asset may occur in isolation, in that the bid-taker and the bidders are not interacting currently or prospectively with respect to this or other assets. With this exception, however, auctions, like bananas, tend to come in bunches. Auto auctioneers try not to schedule sales more frequently than will lead to offering at least 50 or so cars for sale on any occasion when they encourage bidders to congregate. Sotheby's and Christie's tend to collect consignments until they can offer a day's worth of sale of related artwork, and as a result to be auctioning some categories of artwork perhaps only twice a year. Sequential sales of many related items or lots (or of choice among the currently unsold lots in a specified selection) are the archetypal use of oral auctions. Simultaneous sales are usually sealed-bid auctions, whether the assets being sold are identical (e.g., Treasury notes) or differentiated (offshore oil tracts).

Auctions are normally related to other transactions including those not involving auctions. Where bidding is used, even when the asset is being auctioned due to its uniqueness, there is an underlying dynamic: if not this bid-taker, some bid-taker will be selling or seeking a similar asset in the near future. As closely related auctions proceed over time, bidders tend to recur as overlapping subsets of a set of "regulars," often representatives of the firms in a given industry. Because the asset exchanged in an auction is usually an intermediate product or a capital good, amounts bid typically reflect upstream or downstream prices, and the price determined in an auction may affect subsequent upstream or downstream prices.

2.2. Why Is Bidding Used?
Auctions are just one of many ways that society can allocate or transfer assets. The noneconomic possibilities include lotteries and claim staking. The economic ones, aside from auctions, include posted prices and negotiations. (For a fuller discussion of the possibilities, see Shubik, 1970.) Posted prices probably have the lowest transaction costs among the economic alternatives for transferring assets. They are common in many situations, especially in retail sales and in transactions of small size. However, they require the seller to know what price to post. Ignorance of what price to post is a reason for negotiating or holding an auction. The widespread use of negotiations by private firms as a means of transferring assets, including ones similar to those
auctioned by government bodies (e.g., oil drilling rights), suggests that negotiations are a more efficient means of transferring complicated assets than auctions and produce no worse results for sellers. So while auctions can harness competition in various situations, classic economic arguments regarding efficient exchange and revenue maximization may only infrequently explain the use of auctions.

One critical reason for the use of bidding is that the formality of the auction process provides legitimacy in a way that the other economic means cannot. A selling agent can be questioned as to why he posted or agreed to a particular price or why he chose to negotiate with a particular party rather than another. These issues cannot arise if the seller has held a public auction. See Smith (1990) for a fuller discussion and quotes from sellers such as the following from a tow-truck operator:

Sure, the city would probably end up with ten to fifteen percent more if the cars were sold one on one. But you couldn’t do it. There’d be too many problems. There are a lot of regulars here. If I sold a car to one guy, another guy might come in the next day and say “Hey, I hear you’ve got a 1983 red Caddie.” I’d answer, “No more. Vinnie bought it yesterday.” Right away he would start to bitch. “What did you sell it to him for?” “A grand.” “A grand?” he’d answer. “Hell, I would have given you that much. You know I have a thing for Caddies. It’s not right, you should have given me a call.”

It is just not worth it. The next thing, someone from downtown would be calling to find out if there was monkey business going on and if I was selling cars for less than they were worth.

Bidding is fair and, equally important in political contexts, it is widely perceived as fair. It may make legitimate a transfer that would otherwise be suspect. For example, Dam (1976) argues that a 1974 British tax on North Sea oil was a direct consequence of the failure of an earlier government to use auctions in transferring North Sea oil tracts to the private sector.

Notice that, as the above quotation illustrates, bidding also has the effect of depriving a selling agent of power. This can be important when the selling agent is not completely trusted. It can be argued that Congress requires that Federal oil tracts be auctioned because it trusts neither political appointees nor civil servants to deal with oil companies over such transfers.

Bidding is also used in forced sales such as bankruptcies. In forced sales, the seller cannot wait for someone to accept a posted price and cannot say no to offers, a necessary ability for there to be meaningful negotiations. Milgrom (1987) considers a seller in a weak bargaining position. Out of a wide abstract specification of feasible selling mechanisms, he finds that conducting an auction without a reserve price is an expected-revenue-maximizing mechanism.

2.3. Who Chooses Auction Type, and Who Decides to Participate

While sometimes a seller or a seeker of an asset conducts his own auction, the vast bulk of private auction exchanges are consigned to an auctioneer serving as intermediary. Of course, the sale or acquisition is consigned with some instructions. In auctions with non-trivial reserve prices, it is almost unheard of for the auctioneer to be the one determining the reserve price. However, it is the auctioneer as agent, not the principal, who makes most of the decisions about auction form. For example, while the principal may set the reserve price, the auctioneer typically makes the decision as to whether reserve prices in his auctions are publicly announced or kept secret and, indeed, whether to admit that bidding has ceased below the reserve price auction, or to “hammer down” the asset as if a buyer existed (and quietly return it to its owner).

In some auction markets—e.g., US Treasury financings, fine art and bloodline yearlings—potential buyers as principals contract with independent agents to bid on their behalf. It is more common, however, for the bidder to be acting on his own or his employer’s behalf.

Bid preparation costs for mineral exploration lease auctions and major construction project auctions run into the thousands of dollars. Not all auctions create such a serious barrier to competing, but participation as a bidder, or even making a reasonably informed decision as to whether to compete, is never costless. There are many people who find attending auctions interesting, and frequently distributors or brokers who consider the information gained in observing bidding valuable, but neither principals or auctioneers can count on these motivations to create competitive bidding. Bidders in most auctions are there to profit from bidding and winning; when many competitors means they have a smaller chance of winning, their participation is depen-
dent upon a sufficient expected profit in the event of winning to justify incurring participation costs.

2.4. Interrelations with Other Transactions
All parties in an auction are likely to adapt their behavior to the outcomes of previous transactions. A seller will adjust his reserve price based upon the information garnered from previous sales of related assets, by him and by others, upon the effects of his own previous sales on his inventory, etc. Similarly, a bidder will adjust his willingness to pay for an asset and perhaps even his decision to participate in a sale based upon his interpretation of prices fetched in previous sales, and upon the effects on his inventory of his own purchases and unsuccessful bids. This latter impact is particularly important in a sale consisting of sequential auctions of perhaps dozens of related assets. When many assets are sold simultaneously by sealed bids, the bidders cannot adjust willingness to pay for an asset based upon success of other bids in the same sale.

Bidding behavior is also affected by anticipation of the impact of that behavior on future transactions. Within a sale, a bidder bidding aggressively on one of the assets sold early in the day may signal to rivals a greater likelihood that he will be bidding aggressively in later auctions. If rivals interpret the reason for his higher willingness to pay as due to a unique aspect of the current asset, it may not affect their later behavior. If they conclude he wants to win many auctions because of an unusually low inventory, they may lose interest in competing aggressively. However, if they draw the inference that he has information that the resale value or marginal productivity of assets of this type has just risen, they may respond by bidding more aggressively.

Transactions occurring after a sale will typically be influenced by the outcome of the sale, and anticipation of these influences will alter bidding behavior. For example, a potential cogenerator bidding for an electricity supply contract realizes that, if successful, he will be negotiating with contractors, labor unions and others. As discussed below, the terms they demand may be related to the price he will be paid. (Even an unsuccessful bid on this project might reveal information that others will seek to use advantageously after a successful bid next time.) Similarly, prices in resale markets will naturally react to prices paid in auction markets.

3. The Mainstream Theory of Single Isolated Auctions
The bulk of the analytical literature on auctions deals with models of individual auctions. In these models, one or all of the bidders attempt to maximize their expected profit (or their utility) in the single auction. Occasionally, this is a realistic assumption. In many other important situations, it is clearly incorrect. However, single auction models can sometimes serve as useful building blocks or starting points for analyses that take account of the effects of other transactions.

3.1. Decision Theory Models
The earliest bidding models (See Friedman 1956, 1957) assumed or calculated a probability distribution, \( F(x) \), for the best competitive bid, \( x \), and then had the bidder choose the bid, \( b \), that maximized his expected profit, \( (v - b)F(b) \), relative to a known expected value, \( v \), for the auctioned asset. (If \( v \) is not known, then its expectation may be substituted for it provided \( v \) and \( x \) are independent variables.) Where \( F(x) \) was calculated, it was usually calculated as the distribution of the maximum of independent draws from the bid distributions of each competitor. Such models have been used by many bidders, particularly for Federal oil leases in the 1960s and, in some cases, even later.\(^1\) One characteristic of such models is that they lead to more aggressive optimal bids when the level of competition is increased.

In a key paper, Capen et al. (1971) argued for an alternative approach. They attacked the assumption, implicit in the models of Friedman and most of his many followers, of statistical independence between the best competitive bid and the value of the asset being sold. They argued persuasively that, especially in the context of oil lease bidding, the best competitive bid could be a good predictor of the asset’s value. Instead, they proposed a “common-value model” in which the asset value was the same for all bidders, but unknown to them. They assumed all bidders make statistically independent estimating errors. Hence, bidders who make unbiased estimates of asset value will tend to be disappointed in the value of what they win, especially if the competition is heavy, since they will tend to win auctions in which their estimate was optimistic and lose

\(^1\) See Keefer et al. (1991).
ones in which it was pessimistic. Bidders who do not correct sufficiently for this selection bias will be impoverished by what Capen et al. termed "the winner's curse." This selection bias correction needs to be greater and, hence, leads to less aggressive bidding when competition increases (beyond two serious competitors) or when estimating accuracy decreases.

Oren and Williams (1975) demonstrated explicitly that in a wide class of common-value auction models, the expected value of a bidder's value estimate given that he wins the auction is larger than his unconditional expected value. The conclusion that a bidder must treat his estimate as more optimistic upon learning that his bid won does not depend upon significant assumptions about symmetry, estimating bias, bidding strategies or auction type.

Stark (1974) analyzed selection of a set of unit prices for a single contract, and showed that the solution of a simple linear programming problem maximizes expected profit. Stark's concern was road building contracts, and the unit prices covered items such as cubic yards of earth moved and number of signs. However, his analysis is directly relevant to federal timber bidding where the unit prices are for board feet of various species of timber. When bidders use the bids generated by the linear program, the result is "skewed" or "unbalanced" bids in which the unit prices are adjusted as much as possible to take advantage of differences between the way the price will be viewed by the bid taker and by the bidder. Differences in quantity estimates as well as in the timing of payments are exploited. Diekmann et al. (1981) have generalized Stark's result, getting a quadratic program to be solved by risk averse bidders.

Rothkopf (1991) develops formulas for optimal strategies for bidders who can submit two or more bids in an auction and then, after the bids are opened, withdraw any bid not necessary in order to win, perhaps paying a withdrawal penalty. The formulas depend upon the number of bids that are possible, the size and nature (i.e., fixed or proportional) of the penalty, if any, for withdrawing a bid, and form of the distribution for the best competitive bid.

3.2. Game Theory Models
We now consider mainstream game theory models of single auctions. In such models:

1. There is a single, isolated auction involving a fixed set of bidders;
2. There is a symmetric Nash equilibrium (i.e., every bidder optimizes his behavior with respect to the strategies of all other bidders) based on assumed ex ante symmetry in the distribution of private information;
3. The rules of the auction are commonly known, firm and credible; and
4. There is common knowledge of the distributions underlying private information.

The earliest game theory literature (beginning with the seminal paper by Vickrey 1961) makes three further "benchmark" assumptions:
5. Bidder's private signals about asset values are independently distributed;
6. The asset's value to a bidder is independent of the private signals of other bidders; and
7. The bidders are risk neutral.

The oft-studied "independent private values model" employs both Assumptions 5 and 6. Several interesting results hold under these assumptions. First, in an English auction, each bidder has a dominant strategy (i.e., optimal strategy no matter how rivals compete) in which he continues to compete up to his value. When all bidders follow such strategies, an efficient outcome is guaranteed: the item for sale is sold to the bidder valuing it most for a price at or marginally above the second-highest value attached to it by any bidder.

A similar result holds for sealed second-price auctions. This time it is the optimal bids rather than the "strategies" that equal the bidders' values. As with oral auctions, the item goes to the highest valuer at the second-highest valuer's value.

Dutch and standard sealed bidding call for identical strategies in which, in line with the decision theory approach for independent private values discussed above, the bidders select bids by balancing the risk of losing against the profitability of winning. Since the symmetric equilibrium bid function is increasing in the bidder's private valuation, the sale is made to the highest valuer.

Myerson (1981) (see also Riley and Samuelson 1981, Harris and Raviv 1981) demonstrates how constrained a seller's options to influence revenue are within the benchmark model. He shows expected revenue equivalence for any two auction mechanisms that have the properties that (1) the asset always goes to the bidder
with the highest value, and (2) every bidder would attain zero expected profit if his value were at its lowest possible level. In particular, this implies revenue equivalence for the four standard auction types. Myerson also shows that an expected-revenue-maximizing auction uses any of the four auction types with the same suitably chosen reserve price, determined by a formula which depends on the distribution of private values, but not on the number of bidders.

Revenue equivalence results are not robust. They require the special assumptions of independence of private information, symmetry and risk neutrality. Moreover, no general ranking of first-price and second-price expected revenue is robust to changes in these assumptions. (See Vickrey 1961, Griesmer et al. 1967, Maskin and Riley 1983.) Second-price procedures, both oral and sealed, continue to guarantee that in equilibrium the item sold always goes to its highest valuer, but first-price procedures no longer do so.

Now we relax the benchmark assumptions. First, with risk averse bidders who know precisely their value for the asset, expected revenue is greater with first-price bidding than with second-price bidding. (See Holt 1980, Harris and Raviv 1981, Maskin and Riley 1984.) However, this result is less widely applicable than it first seems. In the independent-private-values context, the only risk assumed to be facing bidders is the risk of losing the auction; hence, risk-averse bidders are not cautious, but anxious to win.

Next, we relax the assumption of independent private information. Milgrom and Weber (1982) have defined a class of value functions they called “affiliated values.” This class has independent private values as one extreme case and common values as another. They showed that when bidders’ value estimates are affiliated, expected revenue in an oral auction in which bidders can observe each other drop out is at least as great as that in a sealed second-price auction. The expected revenue from a second-price auction is at least as great as that from a sealed bid auction, which in turn remains strategically equivalent to a Dutch auction. Lack of statistical independence and a bidder’s lack of certainty about asset value each play a role when the independent-private-values assumption is relaxed. Table 1 gives more details about the effects on bidder strategy and on expected revenue.

Of particular interest are results on the role of information. Milgrom and Weber consider the possibility that a seller has access to information which is correlated with asset value, e.g., an independent appraisal of some work of art for sale, or geologic data on an offshore oil tract. Among many information-revealing strategies—e.g., revealing only favorable, or only incomplete, or

### Table 1

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<th>Bidders Have No Residual Value Uncertainty</th>
<th>Statistical Independence</th>
<th>Semi-strict Affiliation</th>
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<td><strong>English, 2nd-Price Strategies: Dominant Strategy to Bid Value</strong></td>
<td><strong>Equilibrium Expected Revenue: Complete Revenue Equivalence</strong></td>
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*Source: Harstad and Rothkopf 1991.*

"Semi-strict Affiliation" is affiliation without statistical independence in any subset of variables. "M-W English" refers to the version of the English auction analyzed by Milgrom and Weber in which bidders can observe the point at which each of their competitors drop out of the bidding. References to first-price auctions apply to both standard sealed bidding and Dutch auctions.
only noisy information—the seller maximizes expected revenue in any standard auction form by fully revealing all such correlated information. (Bidders are assumed to know the bid-taker’s information policy.) In a similar vein, Riley (1988) shows that tying revenue to information about asset value that becomes available ex post, as when a part of revenue is raised via royalties, enhances revenue.

Wilson (1979) analyzes common-value auctions. His principal result is about the information revealed in the winning bid in a first-price auction. It might seem that this could not be more information than was available to the single bidder who happened to make the high bid, based upon his own imperfect estimate of asset value. However, the high bid also implies an upper bound on the estimates observed by losing bidders, which, as Wilson finds, is highly informative. As the number of bidders approaches infinity, under some limiting assumptions, the high bid converges to the unknown true asset value. Milgrom (1979) obtains necessary and sufficient conditions for nearly the same result.

The privacy of his information in a common-value auction is important to a bidder. Milgrom (1981) considered a bidder whose private information was a “garbling” of other bidders’ information, in that his information is independent of asset value once conditioned on other bidders’ information. In a second-price auction, such a disadvantaged bidder has no strategy available that yields a positive expected profit. An interesting case arises if two of three bidders both observe the same relatively informative estimate of asset value, while the remaining bidder privately observes an inferior, less informative signal. The two bidders with the same superior information each have zero expected profit while the bidder with inferior but private information makes a positive expected profit. (Related results are in Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom and Weber (1983).)

Equilibrium bid functions for sealed second-price and English auctions are conditional expectations. They can yield closed-form expressions for sufficiently simple underlying distributions. Game theoretic bidding models of first-price auctions usually yield differential equations that must be solved numerically. However, there are a few interesting exceptions that yield analytic formulas. One of these is for a game involving n bidders drawing independent private values from a uniform distribution on [0, 1] and then submitting standard sealed bids. Vickrey (1961) showed that it is a Nash equilibrium strategy for each bidder to bid \( b(v) = v(n - 1)/n \), where \( v \) is the bidder’s private value. Before drawing a value, each bidder has probability \( 1/n \) of winning the auction. The average profit of the winner is \( 1/(n + 1) \). After drawing value \( v \), a bidder has probability \( v^{n-1} \) of earning a profit of \( v/n \). The bid-taker’s expected revenue is \( (n-1)/(n+1) \).

Analytical solutions can also be obtained in a particular class of common value auctions when bidding strategies are restricted to multiples of estimated value. (Multiplicative strategies may be behaviorally reasonable; models based upon them may provide superior explanations of auction data (see Paarsch 1991); they may approximate optimal unrestricted policies especially in the limit as the amount of common prior information becomes small, but there is no useful model for which they are exactly optimal. See Rothkopf 1980a, 1980b; Engelbrecht-Wiggans and Weber 1979a.) Rothkopf (1969) showed that when \( n \) bidders estimate a common cost, \( c \), with estimates, \( c_i, i = 1, 2, \ldots, n \), drawn independently from a Weibull probability distribution\(^2\) with density \( f(c_i) = a mc^{m-1}e^{-ac} \), where \( m \) is the parameter that controls the standard deviation to mean ratio of the distribution and \( a \) is chosen so as to make the estimate unbiased, i.e., \( a = [\Gamma(1 + 1/m)]/c \), the equilibrium multiplier is given by

\[
P^* = m(n - 1)n^{1/m}/[m(n - 1) - 1].
\]

When all bidders use this strategy, the expected profit of each bidder is given by

\[
E = c / \{n[m(n - 1) - 1]\}.
\]

The multiplier \( P^* \) takes its integer minimum for \( n = 3 \). Beyond that, the increased competition produces increased allowance for the winner’s curse. Despite this, the winning bidder’s expected profit declines approxi-
mately as the square of the number of bidders and decreases when estimating accuracy increases.

In addition, Rothkopf (1969) presents a mirror image highest-bid-wins model in which the value estimate is drawn from a Gumbel distribution. Rothkopf (1980b) also gives analytic results for equilibrium strategies for a case in which bids are restricted to being a general linear (i.e., affine) function of the estimate.

The game theory literature contains a number of papers containing results on the design of "optimal," i.e., revenue maximizing, auctions in which the bidder may set a reservation price and even charge fees to losing bidders. (See for example Laffont and Maskin 1980; Harris and Raviv 1981; Myerson 1981; Riley and Samuelson 1981; Matthews 1983, 1984; Maskin and Riley 1984; Moore 1984.) However, the results so far seem to be of theoretical rather than practical significance. The cited models do not seem to be realistic enough for most practical purposes. In addition, the "optimal" auctions are usually quite complex, and there is no evidence for their use in practice.

However, game theory models of proposed alternative sales processes have, at times, proved useful as has the use of game-theory-based thinking in models that are not pure games. See for example Wilson’s (1979) game-theoretic model of auctions of shares, McGuire’s (1978) proposal for "interact" competition for coal leases and the simulation of electricity purchase auctions by Kahn et al. (1990).

4. Enrichment of the Mainstream Theory

4.1. Participation Decisions

The costs of preparing a bid, including the costs of obtaining the information that goes into bid preparation, are normally sunk costs when a bid is submitted. They are incurred when it is decided to prepare a bid. Typically, this decision is made on the basis of the expected profitability of bidding, which in turn requires attempting to anticipate the participation decisions of other potential bidders (in general, fewer rivals will mean a greater probability of winning and greater expected profitability in the event of winning). It is common to seek the bid-taker’s help in anticipating the level of competition; refusing to provide information he possesses may not necessarily be in bid-taker’s interest. In some auctions such as oil lease sales and new stock offering underwritings, the decision expands to include participating separately, negotiating a joint bidding and information pooling syndicate, or foregoing bidding entirely.

Formal decision-theoretic treatments of the decision to prepare a bid for a single auction date back to Flueck (1967) and Engwall (1976) and are unsurprising. Simmonds and Slatter (1978) discuss a firm’s broader decision of how large a staff it should have for preparing bids and, thus, how many of its bidding opportunities it should accept. Engelbrecht-Wiggans (1987a) discusses the bid-taker’s large incentive to attract bidders.

Harstad (1990) presents a game theoretic model of a common-value auction in which bidder participation is determined endogenously rather than predetermined. He finds symmetric Nash equilibria in which a large group of potential bidders select a probability of participating, by privately observing an informative estimate of the common value of the asset (at a cost) and submitting a bid. The expected number of bidders increases until it is no longer profitable for bidders to raise their participation probabilities. Expected revenue is then equal to the expected value of the asset less the aggregate expected level of bidders’ participation costs. Hence, somewhat surprisingly, a smaller expected number of bidders in advantageous to the bid-taker.

Hausch and Li (1993) consider a special example that also entails endogenous decisions on information acquisition.

Harstad (1993) extends this approach to the General Symmetric Model of Milgrom and Weber (1982), considering bidder participation decisions when there may be both private-value and common-value aspects to asset value. The characterization of expected revenue is extended, but now the inference is that attracting fewer bidders will yield expected revenue which is closer to the expected value of the asset to the bidder valuing it most highly. However, this highest expected asset value may be lower when fewer bidders are attracted to the auction. So the parameter space of auctions is divided into "overattractive" and "underattractive" regions, according to whether the increase in highest expected asset value with more bidders is sufficiently strong to overcome the seller’s receiving an expected revenue.
further below this expected value. Thus, the mainstream revenue rankings for an exogenous number of bidders are upheld in the overattractive region, but reversed in the underattractive region. This accords with observed practice, where some auction design decisions are consistent with the mainstream revenue rankings, but patterns of contrary auction design decisions can also be found. The endogenous participation aspect of this model results in a socially optimal balance between the efficiency benefits of having additional bidders and the additional costs of bid preparation.

Hausch and Li (1991) consider game-theoretic models of independent-private-value auctions in which the bidders decide how much costly information to gather. They find that when the accuracy of the information a bidder acquires is private to that bidder, the revenue equivalence of first-price and second-price auctions holds, but that when that accuracy is publicly observable, revenue equivalence fails.

Symmetry is at least as restrictive an assumption in these models as in the mainstream theory. In principle, Harstad’s (1993) model can be adapted to asymmetry, but the resulting equations seem impenetrable. Another class of participation decisions largely ignored is the consignment decisions of potential sellers. Treating the seller and the auctioneer as a single party can mask a significant principal-agent problem.

4.2. Information Revelation

4.2.1. To Other Bidders. A bidder’s estimate of the cost of a construction project is likely to reflect details of the way in which his firm processes information about the specifications of the building to be built, architectural and site-specific issues, and also details of his own firm’s production technology. The possibility that some of the asset-specific information this firm has gleaned might be inferred by rivals from its bidding behavior, to its disadvantage in the current auction, arises only in oral auctions. By the time rivals infer this information from a sealed bid, bids have been opened. However, a firm has concerns about revealing information common to oral and sealed bidding. Rivals who learn something about its production technology, or its estimating methods, from its bid in this auction, may then be in a position to predict better its bids on future projects. Firms worry about disclosing information to rivals that will create disadvantages during the current auction, and they may go to great effort to withhold private information which is key to future bidding profitability.

As discussed below, Rothkopf et al. (1990) show that second-price auctions no longer have dominant strategies with bids equal to private values if bidders wish to keep rivals from learning their private information. In oral auctions, Harstad and Rothkopf (1991) have noted that the capability of bidders to communicate privately with the auctioneer (via a slight nod, prearranged hand signal, etc.) calls for a richer model of English auction strategies than Milgrom and Weber (1982) incorporate. If bidders’ privacy in communicating with seller could be completely maintained, expected revenue matching that of a second-price auction should result. If a bidder’s rivals can discern part of his private communications with auctioneers, expected revenue is intermediate between that of a second-price auction and the Milgrom-Weber formulation.

4.2.2. To Third Parties. A straightforward example of the problem of information revelation to third parties arises in electrical power purchase auctions. Often, several industrial firms have plans to build cogeneration facilities, typically with quite diverse production technologies. They bid to supply surplus electrical power to a utility, and must follow a successful bid with negotiations with third parties necessary to complete the transaction (contractors, labor unions, local governmental units who must give permission, etc.). Rothkopf et al. (1990) analyze this situation. They use an independent-private-values model, under second-price rules, assuming that third parties capture some fraction of the rent, i.e., the excess of the price to be paid over the winner’s offered price. The rent capture interferes with the dominant strategy argument. Even if two different bids would still win, the lower will reveal more rent and thus yield less post-capture profit. The equilibrium exhibits a “profit equivalence” that follows Myerson’s argument (1981) on revenue. Hence, on average the rent capture by the third parties comes at the expense of the bid-taker. (See also Engelbrecht-Wiggans and Kahn 1991.) Motivated by the concern about the effect of information revelation, Nurmi and Salomaa (1993) have suggested a cryptographic version of the sealed second-price auction to limit the information revealed.
In many situations, there will be correlated uncertainties for bidders, e.g., prices in resale markets, making the private values assumption suspect. In such situations, a second-price auction would still yield a transparent difference between the winning bid and the price, but this difference would not be the expected rent, although it would be correlated with it. Its influence on post-auction bargaining would be hard to model.

Waehrer (1992) considers the interaction between competitive bidding to sell equipment and the winning bidder’s market power in subsequent spare part sales.

4.3. Interdependencies

4.3.1. Internal to Bidding Firm. Suppose a number of identical assets are being sold simultaneously in a private-values setting. If each bidder desires only one item, then a second price auction still yields bids equal to values as a dominant strategy. However, any bidder who desires more than one item and submits multiple bids has an incentive to bid less than his value for any item other than the first. This incentive arises because there is some chance that his bid on an item will set the price for another for which he has offered to pay the same or a higher price. Dubey and Shubik (1980) have proposed a sale mechanism involving differentiated prices that restores incentive for truth revealing bids, essentially by paying off buyers for the monopsony power inherent in their having more than a marginal demand.

In general, the guidance from game theory for the revenue-maximizing choice between discriminatory and nondiscriminatory sales of identical items is unclear. On the one hand, with affiliated values, symmetry and no risk aversion, the uniform price auction will yield at least as much expected revenue. On the other, with independent private values, symmetry and risk aversion, the discriminatory auction will yield at least as much revenue. (See Weber 1983.)

A number of papers dating back to Friedman (1956) have considered the situation faced by bidders who wish to bid on more than one of a number of differentiated items that are to be sold with simultaneously opened bids but who face a constraint upon the total of their bids. This is a situation often faced by oil company executives preparing bids for offshore oil tracts. Stark and Mayer (1971) pointed out that the problem of optimally allocating the scarce resource of bidding dollars over the competing opportunities provided by different sales did not have a concave objective function and, hence, could not be solved in general by means of marginal adjustment of proposed bids. They provided a computationally tractable dynamic programming approach for solving the problem. If there are \( n \) simultaneous auctions, \( E_i(B_i) \) denotes the expected profit in auction \( i \) with bid \( B_i \), \( V_i(B) \) is the maximum expected profit that can be obtained from auctions 1 through \( i \) with a total bid limit for these auction of \( B \) and where a bid of 0 is understood to mean no bid at all, the relevant dynamic programming recursion is given by

\[
V_i(B) = 0, \quad V_i(B) = \max_{0 \leq B \leq B_i} \left[ E_i(B_i) + V_{i-1}(B - B_i) \right],
\]

\( i = 1, 2, \ldots, n. \)

Rothkopf (1977) showed that a modified marginal adjustment process could obtain a bounded, nearly optimal allocation of bidding dollars when the expected return from each auction as a function of the amount bid had a typical form involving an initial convex region followed by a concave one. He also showed that at most one auction could have an optimal bid in the convex region and gave conditions under which there cannot be any. Engelbrecht-Wiggen (1987b) considers a buyer who uses an agent to bid for him in simultaneous auctions. The agent’s maximization of the total value of property won\(^3\) subject to a constraint on expected expenditures is equivalent to maximization of expected profits subject to the same constraint. However, a constraint on expected expenditures is not workable as a control mechanism when the probabilities in the expectation are the subjective probabilities of the agent. A more workable constraint for the buyer, one on total bids, does lead to different behavior.

\(^3\)Engelbrecht-Wiggen and Weber (1979b) have solved a particular, game-theoretic model in which bidders participate through agents in \( n \) simultaneous oral auctions for identical items. Each bidder is assumed to have value for only one item; any items beyond one bought are worthless to their winner and not resalable.

This is sometimes used as a characterization of oil company exploration vice presidents. For modeling along these lines, see Arps (1965).
Smith and Rothkopf (1985) consider the situation faced by a bidder bidding in simultaneous auctions who will incur a single fixed charge if and only if he wins one or more of the auctions. They present a single-state-variable dynamic programming formulation, applicable when the auctions are statistically independent, and computationally helpful conditions for the general case. Cohen and Loeb (1990) consider the accounting implications of a similar situation.

Kortanek et al. (1973) consider models in which bidders are bidding in a sequence of auctions to sell products that use common scarce resources, such as production capacity. While the particulars differ from model to model, the optimal strategy can always be characterized by the relationship:

\[ \text{bid price} = \text{direct cost} + \text{opportunity cost} + \text{competitive advantage fee}. \]

There is little literature on the realistic problem of a bidder bidding in a series of auctions in order to replenish inventory.

4.3.2. External to Bidding Firm.

4.3.2.1. Effect on Rivals’ Future Bids. A bidder’s choice of bid in an auction depends upon the anticipated aggressiveness of rivals’ bidding. When bidding in previous auctions provides indications of how aggressively rivals have bid, the bidder’s bidding in this auction will react to these indications. Now, switching perspectives and looking forward, a bidder’s bidding in this auction will be scrutinized by his rivals to discern clues as to how aggressively he will bid when they compete with him in upcoming auctions. In general, when this effect is incorporated, the bidding behavior that maximizes expected profit in one auction viewed in isolation may well not be consistent with long-run expected-profit-maximizing behavior.

In game theory parlance, what results is a repeated game. The extensive literature on repeated games is of little use to competitive bidding analysts, for three reasons. First, nearly all analyses of repeated games assume that at any point, a player has only finitely many (pure) actions available. Both game-theoretic and decision-theoretic models of auctions and bidding assume that a bidder can select any real number. While bids are typically a multiple of a smallest money unit, the infinite choices approach has a number of advantages over a finite-bid-level approach. These include the existence of pure strategy equilibria and, importantly, the ability to use existing auction models as submodels. Second, the repeated game literature assumes that precisely the same “stage” game is played over and over, while auctions tend over time not to be exact duplicates. For example, governments tend to auction off more desirable oil tracts first; and information from exploration of previously sold tracts affects estimates of the values of tracts sold later. Third, unlike typical repeated game models, repeated auctions do not usually involve precisely the same sets of bidders each time. For example, an art gallery may sit out one sale, but compete in the next auction of a slightly different kind of art.

Oren and Rothkopf (1975) consider the effect of bids on future competitive behavior. In their model, a scalar characterizes the state of competition and a “reaction function” relates a bidder’s bid to changes in that state. They provide a general dynamic programming approach to finding the optimal bidding strategy and apply it to Rothkopf’s 1969 multiplicative strategy equilibrium model described above. Note that because they use a particular reaction function, their model is not a dynamic game, but rather a static game in which each bidder takes into account his estimate (which is not necessarily correct) of the effect of his present bid upon the future behavior of his competitors. One reaction function they used is

\[ Q_i(t + 1) = Q_i(t) + \alpha p_i(t)[P_i(t) - Q_i(t)], \]

where \(P_i(t)\) is bidder \(i\)'s multiplicative strategy in auction \(t\), \(Q_i(t)\) and \(Q_i(t + 1)\) are the strategies used by bidder \(i\)'s competitors in auctions \(t\) and \(t + 1\), respectively, \(p_i(t)\) is the probability that bidder \(i\) wins auction \(t\) (It is a function of \(P_i\) and \(Q_i\)), and \(\alpha\) is a strength-of-reaction parameter. The resulting equilibrium strategies and the expected profits are

\[ P^* = m(n - 1)n^{1/m} / [m(n - 1) - 1 - F] \quad \text{and} \quad E = c(1 + F) / n[m(n - 1) - 1 - F], \]

where \(D\) is the discount factor between auctions and \(F = \alpha D / (1 - D)\) is a factor indicating the effect of the sequential nature of the auction. Note that when \(F = 0\), \(P^*\) and \(E\) revert to the expressions given above. As the strength of the competitive reaction increases and as
the discount factor between auctions nears unity, \( F \) increases causing the bids and the expected profits to increase.

Several recent pure game-theoretic models analyze sequential auctions. These models differ in the nature of the individual auctions (e.g., first-price vs. second-price and independent private values versus common value vs. general affiliated values) and in the relationship between the auctions (e.g., bidders drop out after purchasing one item vs. continued participation, and independent values in successive auctions vs. values in one auction containing information on values in later auctions.)

Hausch (1986) compares selling a group of items using sequential auctions to selling them using simultaneous auctions. He considers a context in which information about the value of one object conveys information about the value of others. There are potential benefits to the bid taker from bidders in later auctions having information from earlier ones, and in some cases this can be the dominant effect. However, in many cases this effect is smaller in equilibrium than a deception effect. Bidders have incentive to bid less aggressively in earlier auctions so as to avoid increasing the subsequent valuations and bids of their competitors. This result was foreshadowed by a simple two-stage, two-object, two-bidder signaling model of Ortega-Reichart (1968).

Weber (1983) reports that in a symmetric, affiliated values model the prices of sequential sales when each bidder wishes to win at most one of the identical items will exhibit a upward drift in equilibrium. The upward drift, which is not a sage empirical prediction, is clearly dependent upon the assumptions of identical assets and information. Engelbrecht-Wiggans (forthcoming 1994) finds a downward drift of prices accompanied by a downward drift in the profits of the winning bidder in a model of stochastically equivalent items.

Bikhchandani (1986, 1988) considers a model in which two bidders participate in a series of second-price auctions. The auctions are common-value auctions except for the existence of a small probability that the “advantaged” bidder has a slightly higher valuation. He finds an asymmetric equilibrium in which the advantaged bidder establishes a reputation for aggressive bidding and thus forces the other to bid quite aggressively in order to avoid the winner’s curse. Due to the second-price nature of the auctions, this equilibrium is quite unfavorable to the seller.

Aggressiveness of bidding in the current auction presumably influences not only the bidding strategy of rivals in future auctions, but also the number of rivals. Thus, there is a so far untaken opportunity to combine dynamic strategy issues with participation issues.

4.3.2.2. Effects on Profitability in Other Markets. Bikhchandani and Huang (1989) have constructed a model of a common-value auction combined with a resale market. They consider the effect in the resale market of information on bids and derive conditions under which uniform-price auctions produce more expected revenue than discriminatory auctions. Greenleaf et al. (1993) consider a model of art auctions in which an auction house negotiates with potential sellers over a guaranteed minimum price and a commission on the excess. In the model, if the item fails to reach the guaranteed price in the auction, the auctioneer buys it at that price and then must sell it in an aftermarket in which its failure to sell in the auction is known and marks the object as “damaged goods.”

4.3.3. Bid-Taker’s Concerns. The decision of what to sell and when to sell it can have a major impact. The “area-wide” leasing of federal offshore oil tracts carried out under Interior Secretary Watt in the early 1980s, changed previous policy in a way that lowered government revenues by billions of dollars. (See Moody and Kruvant (1990) as well as Stiglitz (1984) and Rothkopf and Harstad (1990).)

It is also true that the some of these other decisions may affect the appropriateness of alternative sale mechanisms. For example, if many mineral leases are offered for sale at once, the increased chance that a particular lease will receive only one bid might justify higher minimum bid requirements. Also, if many leases are offered at once, the limited ability of bidders to finance multiple purchases could increase the relative attractiveness of sequential sales compared to simultaneous sales. It could also suggest that contingent bidding schemes might be worth considering.

Other important sale-related decisions include how to charge (e.g., per unit royalty, one time bonus, annual rent, profit share, etc.)\(^4\), who may bid (e.g., citizenship requirements, financial responsibility requirements, joint
bidding bans,\(^5\) etc.), and special advantages (e.g., for small business, minority business, etc.).

Bid-taking buyers desiring to purchase multiple items sometimes split the award, buying some items from one or more bidders asking a higher unit price. This practice has typically been viewed as a (costly) way of improving competition in future procurements (e.g., see Anton and Yao 1987) or occasionally as justified by diseconomies of scale (see Anton and Yao 1989 and Seshadri et al. 1991). However, Klotz and Chatterjee 1991 have developed a model of a single auction in which entry decisions by potential bidders with linear costs and risk aversion combine to make it profitable for a bid-taker to precommit to a split procurement.

4.4. Changing, Bending and Breaking the Rules. While most auction models assume that the auction rules are clear, understood, and universally followed, in reality bidders often look for ways to bend, change or get around the rules. Many a losing bidder has tried to find an excuse to reopen a sealed bid auction with an improved offer after competitive bids have been revealed. Winning bidders have been known to withdraw winning bids in order to allow associates to win with less aggressive bids. (See Rothkopf 1991 both for models and for a report of a particular instance of this.)\(^6\) Sellers using unit price auctions have had trouble defining “unbalanced bidding” and thus making their rules against it effective. The Treasury recently learned that Salomon Brothers evaded the rule against any bidder winning more than 35% of an issue by submitting unauthorized bids in the names of its clients.

Especially in unique situations that otherwise would fit well the isolated auction assumption, bidders may profit by treating the “auction” as the first step in a negotiation process. For example, in 1955 Shell Chem-


\(^6\) Interestingly, Harstad and Rothkopf (1992) show that formalizing the opportunity to withdraw a bid can serve as a form of “winner’s curse insurance,” and that offering this “insurance” can be in the bid-taker’s best interests. See also Waehrer (1993).

\(^6\) The Shell Chemical Company chose to ignore a Congressional mandate to a special Congressional commission to sell off government synthetic rubber plants built during World War II individually by unit. It submitted a single bid for three related units, and when that bid exceeded the sum of the individual bids on the units it was able to persuade the commission to persuade Congress to accept its bid. (See Rothkopf 1983.)

There are a few models of out-and-out cheating in auctions. Comanor and Schankerman (1976) examine the role of identical bids in easing the coordination problem of an ongoing conspiracy to allocate market shares by rigging bids. Robinson (1985) and Graham and Marshall (1987) point out that conspiracies to collude are stable in oral auctions, since the conspirators can observe a violation of the agreed upon course of action in time to react to it, but are not stable in standard sealed bidding. (See also the more complete study of colluders’ incentives in Maiath and Zemsky 1991.)

In some kinds of auctions, bid rigging is common. (See, for example, American Society of Civil Engineers 1985.) Kuhlman (1974), Comanor and Schankerman (1976), Rothrock (1976, 1978), Maltz and Pollock (1979, 1980) and Porter and Zona (1993) have reported on ways of detecting bid rigging from the examination of bidding history. However, the particulars of some of the most used work on this topic are confidential (Rothrock 1991).

Rothkopf et al. (1990) argued that fear of bid-taker cheating (by inserting an artificial or insincere bid between the best and second best bids) contributed to the rarity of Vickrey auctions. Recently, Rothkopf and Harstad (1992) have created models that support that intuition.

4.5. Asymmetries in Evaluation and in Behavior. While asymmetry poses no problems for decision-theoretic models, almost without exception, the game-theoretic literature on auctions assumes either that bidders’ private information is symmetrically distributed, or that bidders have no uncertainty over what the auctioned asset is worth to them. Neither assumption is often met in practice. Salomon Brothers is believed to hold a significant informational advantage in Treasury auctions, where the key issue is the forecast of near-term changes.


6 Interestingly, Harstad and Rothkopf (1992) show that formalizing the opportunity to withdraw a bid can serve as a form of “winner’s curse insurance,” and that offering this “insurance” can be in the bid-taker’s best interests. See also Waehrer (1993).
in interest rates; their advantage stems from the much larger number of bonds they have preplaced with a larger number of clients, most at prenegotiated yields. Some offshore oil tracts are offered for sale at a time when one bidder has previously drilled on an adjacent tract. Even in construction project auctions, a bidder who has dealt with this client and this architect before is in a better position to estimate the likelihood of later design changes, the costliness of responding to them, and the likely reaction to a post-auction proposal to increase compensation in response to requested design changes.

Even when all aspects of the auction, including information, are ex ante symmetric, the assumption of symmetric behavior may be untenable. Because informational symmetry is so uncommon, the principal examples come from laboratory experiments (see §4.6), in which information symmetry can be controlled by design. In a wide variety of experiments in symmetric settings, the null hypothesis of symmetric behavior can be overwhelmingly rejected.

The existence of asymmetric equilibria in informationally symmetric situations has been studied. Maskin and Riley (1991) prove that no asymmetric equilibrium exists in a fairly general symmetric first-price auction model. There are asymmetric equilibria in second-price auction models (one bidder bids impossibly high and all others bid zero), but they appear implausible. Under reasonable conditions, Bikhchandani and Riley (1990) show the nonexistence of asymmetric equilibria in common-value auctions which give each of at least three bidders positive probabilities of winning. They show, however, that n-bidder English auctions, as modeled by Milgrom and Weber (1982) with full information release, are prone in equilibrium to asymmetric behavior by the last two bidders competing.

More important than studying asymmetric equilibria in symmetric models is modeling asymmetric situations. For asymmetric equilibria generated by asymmetric risk tolerances in an otherwise symmetric situation, Harstad (1991) shows that weaker versions (due to multiplicity of equilibria) of common-value equilibrium comparative statics can be generated for second-price rules. First-price auctions yield no similar asymmetric comparative static results.

When the asymmetries underlie asset evaluation, equilibrium behavior in second-price, private-values auctions is unaffected. Rothkopf's 1969 paper that models sealed bidding with multiplicative strategies includes an analytic solution for two bidders with known relative values and numerical solutions for more general asymmetric relative values. In this model, the relative estimating accuracies are symmetric. Harstad (1989) models a three-bidder second-price auction that modifies a common-value setting by giving a single bidder a known relative cost advantage. He is able to characterize an equilibrium for the uniform distribution case. His point is that ex ante knowledge of whether the advantaged bidder is participating makes participation of the other two bidders less likely. As a result, the seller is more likely to receive only the reserve price.


4.6. Models and Economic Laboratory Testing. Partly because of difficulties with econometric measurement, economists have begun doing laboratory experiments. See Plott (1982), Smith (1982, 1987), Kagel (forthcoming 1994) and Davis and Holt (1992) for discussions of the role of laboratory experimentation in economics and Roth (1988) for a methodological critique. In most of these experiments, human subjects are given the opportunity to earn money by making decisions in a real but simple "economy." Many of these experiments have involved competitive bidding.

Some of these experiments are designed to check the predictive properties of Nash equilibrium solutions to game-theoretic models of auction behavior. Often, the experimental subjects converge upon the equilibrium with great speed. See for example Smith (1967) and Plott and Smith (1978). On the other hand, Kagel and Levin (1986), Kagel et al. (1992) and Lind and Plott (1991) were able to find a particular kind of persistent disequilibrium behavior even when they repeated their experiments using experienced businessmen as subjects (cf. Dyer et al. 1989). Experimental control has made
it possible to observe behavior under nested informational structures (Kagel et al. 1987).

Some of the bidding experiments are designed not to test mathematically precise theories, but rather to test proposed new market mechanisms. Grether et al. (1979, 1981) and Rassenti et al. (1982), for example, ran experiments to test proposed schemes for auctioning off landing rights at congested airports and Banks et al. (1989) tested innovative mechanisms for allocating and pricing a planned space station.

5. Use of Bidding Models by Decision Makers

Bidding models address questions of interest both to bidders and to those who run auctions. One way bidding models could be helpful to decision makers in these activities is to be used directly by them. Alternatively, bidding models could be useful indirectly by providing metaphors useful for thinking about auction issues.

Bidders usually have good reasons for being secretive about how they determine their bids. Nevertheless, it is clear that some bidders in sealed bid auctions are using formal decision-theoretic models to recommend or decide upon bids and to allocate exposure in simultaneous sales. (Capen et al. 1971), Keefer et al. (1991) and private communications have given descriptions of the use in oil lease bidding of models of the kind described above. In addition, Richardson (1991) describes the use of a model that extrapolates past competitive behavior to determine bids in a highly repetitive industrial situation.) However, we are not aware of any bidders using game-theoretic models directly to decide how much to bid in particular auctions. Nor are we aware of bidders in oral auctions directly using formal bidding models. As we write this, however, the FCC is considering extensive comments from auction theorists on alternative procedures to auction valuable radio spectrum rights.

Another area where the need for secrecy interferes with desires for open knowledge is in the detection of bid rigging and estimating its effects. Here too, however, we know of the use of models, regression models in this case, by auction managers (Rothrock 1991).

There is some written material available from government agencies on the design of their auctions and related issues. Analysis is sometimes applied to issues such as what form of auction to use, how to divide what is to be sold into separate items, who may bid, how to charge for what is sold, how to evaluate bids, whether to sell items simultaneously or sequentially, what rules to use to limit unbalanced bids, and what rules to use to decide when to reject bids. (For some references, see Rothkopf (1994, forthcoming).) However, we are not aware of direct successful use of any of the models described above by auction designers. The closest to direct use that we are aware of is the use of arguments based upon Vickrey (1961) by John Jurwitz and G. Vail to convince the California Public Utilities Commission (decision no 86-07-004, July 2, 1986) to adopt Vickrey auctions as the format to be used by California electric utilities in purchasing co-generation power. However, for the reasons discussed above, we believe that this decision was unwise, and furthermore, it has yet to be implemented in actual auctions. In addition, the US Treasury Department's experiments with a uniform-price auction for bond sales in 1973-1974 (see Tsao and Vignola 1980) were clearly influenced by auction theory. However, bond traders' hostility to the auction outweighed potential revenue benefits in the political calculation. Thus, we conclude that the record of direct application of the models described above for auction design is weak.

It is hard to know how to evaluate the use of the models to provide a metaphor or framework for less exact analysis. It is clear that ideas arising in bidding theory such as the winner's curse, revenue equivalence, and the need to protect the privacy of the bidder's information have been used by those thinking about auctions. However, theorists may be unable to claim credit for inventing the ideas in such metaphors. Rather, in many areas auction theory may be attempting to capture and include in formal models ideas that are already clear intuitively to some of those with responsibilities for an-

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7 In September 1992, in the wake of the Salomon Brothers scandal, a new two-year experiment with uniform-price auctions for some Treasury bills began, influenced by analysis reported in Chari and Weber (1992).
alyzing and making auction decisions. (See, for example, the discussion of the OCS auction design in Rothkopf et al. (1986).) We do not wish, however, to demean the valuable role of auction theory in refining the thinking involved in auction design analyses or in communicating insights from such analyses.

6. Conclusions

We have tried to emphasize the gaps between the existing theory and the reality of auctions. There are different ways the theory can be evaluated. One of these is as a source of models for direct application by decision makers. Some bidders in sealed bid auctions are using decision-theoretic models to recommend or decide upon bids. However, as discussed above, we are not aware of any direct use of game-theoretic models. Nor do we feel that the current state of this literature is adequate to recommend such applications. Among the common assumptions of such models discussed above are often suspect for these purposes are symmetry, common knowledge, isolation, fixed number of bidders, and unbending rules. Whether game-theoretic models of competitive bidding can be made realistic and useful enough that bidders will have reason to use them remains to be seen. In this respect, the role of game-theoretic models in analyzing auctions is paralleled by their role in negotiation analysis as reported by Sebenius (1992).

Another way that bidding theory can be evaluated is as providing useful metaphors for thinking and communicating about auctions. At this level, the literature is more useful. The modeling presents a variety of useful, although sometimes misused, examples of causal relationships. However, in our view, bidding theory is still playing “catch up” in the sense that there are phenomena that experienced, thoughtful auction participants are aware of that are not well reflected in its models. Until the catching up is completed, the role of the theory in practical auction design will be limited.

We end our essay with advice. For practitioners, we suggest care in matching the assumptions in models to the situations of interest and patience because creative theorists are at work enriching models of auctions. For theorists, we advise attention to the particulars of how auctions are modeled. Modeling improvements will be of more direct value than use of equilibrium concepts more subtle than Nash’s or new theorems that apply only to oversimplified models. For editors and referees, we suggest that relevance be allowed to compete with elegance for scarce journal space. 

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