Comparative Static Effects of Number of Bidders and Public Information on Behavior in Second-Price Common Value Auctions

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Abstract: Comparative static tests of Nash bidding theory in second-price common value auctions show that bidders fail to respond in the right direction to more rivals and to public information concerning the value of the item. The former provides a clear indication that bidders fail to appreciate the adverse selection forces inherent in common value auctions, while the latter shows that policy prescriptions can fail given out-of-equilibrium behavior. These tests of Nash bidding theory apply to a far wider variety of circumstances than in first-price auctions, so there is less scope to rationalize the failure of the theory.

Key words: auctions, common value, information, laboratory experiments

1 Introduction

In common value auctions the value of the item is the same for all bidders. Before bidding, each of the \( N \) bidders privately observes an informative signal (or estimate) which is a random variable affiliated with the value. Affiliation means that a higher estimate for one bidder makes higher estimates for rivals and a higher asset value more likely. Mineral rights auctions, such as outer continental shelf (OCS) leases, are often characterized as common-value auctions; here the signals represent different bidders' pre-sale estimates of lease value. Any auction of a potentially resalable asset involves a common-value element.

In common value auctions, bidders face an adverse selection problem, as the high bidder is likely to have the highest estimate of the item's value. Unless this adverse selection problem is accounted for in bidding, the high bidder may suffer
from a "winner's curse," winning the item but making below normal profits. Bidders in early outer continental shelf (OCS) oil lease auctions are often thought to have suffered from a winner's curse (Capen, Clapp and Campbell, 1971; Mead, Moseidjord, and Sorensen, 1983; Gilley, Carrols and Leone, 1986). Inexperienced bidders in first-price laboratory auction markets suffer from a winner's curse as well, earning negative average profits (Kagel et al, 1989; Lind and Plott, 1991; Garvin and Kagel, in press). Experienced bidders in first-price laboratory auction markets overcome the winner's curse with relatively few bidders, but succumb again when exposed to the heightened adverse selection forces associated with larger numbers of bidders (Kagel and Levin, 1986). "Sophisticated" bidders drawn from the commercial construction industry behave no differently than student subjects in laboratory auction markets (Dyer, Kagel and Levin, 1989).

In spite of these outcomes, Lind and Plott (1991, p. 344) note, "A major puzzle remains: of the models studied, the best is the risk-neutral Nash-equilibrium model, but that model predicts that the curse will not exist." They go on to comment that "Part of the difficulty with further study stems from the lack of theory about (first-price) common value auctions with risk aversion... If the effect of risk aversion is to raise the bidding function as it does in private auctions, then risk-aversion ... might resolve the puzzle; but, of course, this remains only a conjecture." (Lind and Plott, 1991, p. 344). There is also a lack of theory regarding the effect of asymmetries in bidders risk preferences in first-price common value auctions, which makes it even more difficult to unambiguously analyze real life bidding patterns in these auctions.

From this perspective second-price auctions provide an ideal vehicle to explore bidding in common value auctions. In a second-price auction, the high bidder obtains the item for a price equal to the second-highest bid. Although rarely observed in field environments (Cassady, 1967), second-price auctions permit us to observe item valuation behavior while avoiding many of the strategic issues that complicate the more commonly encountered first-price institution. In contrast to first-price auctions, behavior of risk averse bidders is well understood in second-price auctions with both symmetric risk averse bidders and with asymmetric risk averse bidders. Our experiment shows a strong winner's curse even with full accounting for the potential effects of risk aversion on bidding. More importantly, the existence of robust comparative static predictions in second-price auctions allows us to better understand the mechanism(s) underlying this failure of Nash equilibrium bidding theory.

Experienced bidders in auctions with 4 or 5 bidders exhibit substantial positive profits which are reasonably close to the risk neutral Nash benchmark. However, in auctions with 6 or 7 bidders, the same subjects earn negative average profits, as they clearly suffer from a winner's curse. There is no way to rationalize these negative average profits in terms of a Nash equilibria, whether it involves symmetric or asymmetric bidding. However, to simply reject the theory at this point seems a bit harsh and leaves us with no clear understanding of the mechanism(s) underlying these results. Perhaps bidders are simply miscalibrated relative to the Nash point predictions, so that they satisfy the comparative static predictions of the theory? Perhaps they are still learning, so while moving in the right direction relative to the theory they have

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2 This conclusion is, of course, not without its critics. See Wilson (1992) for an evenhanded review of the results from field data.
yet to achieve the equilibrium outcome? Perhaps the overbidding can be explained by a utility for winning the auction? None of these factors can explain or data.

Overly aggressive bidding in second-price common value auctions cannot be rationalized in terms of bidders’ risk aversion. Rather best responses are less aggressive for risk averse than for risk neutral bidders. Further, individual bidders fail to reduce their bids in response to increased numbers of bidders, a comparative static implication of the theory which holds for both risk averse symmetric and asymmetric Nash equilibria, and even extends to auctions where the strategy profile is not an equilibrium (corresponding predictions in first-price auctions require symmetry and are conditional on risk attitudes and the underlying distribution of information at bidder’s disposal). This provides rather incontrovertible evidence that bidders fail to appreciate the adverse selection forces inherent in common value auctions.

Public information is predicted to rise average revenue in second-price auctions, both in symmetric and asymmetric constant absolute risk averse Nash equilibria (no such extensions of Nash bidding theory are available for risk averse bidders in first-price auctions). However, public information is predicted to reduce revenue in the presence of a strong winner’s curse, a prediction which is consistent with the results of our experiment. Not only does this result have public policy implications, but it also rules out overly aggressive bidding on account of a utility of winning the auction, as the utility of winning should be unaffected by the presence or absence of public information. Finally, assuming symmetry and risk aversion (of any sort), we are able to derive predictions regarding individual bidder response to the type of public information provided in our second-price auctions, which provides insight into whether bidders have accounted for the information implied by the event of winning (no comparable predictions have been obtained for a first-price institution). Although the value of this prediction is mitigated somewhat by the extent of the asymmetry reported in bidding, these data still provide insight into why the revenue-raising mechanism of second-price auctions fails, and the proposition demonstrated may prove useful to those who choose to replicate our results or to test the theory with even more experienced bidders.

The robustness of the predictions of second-price auction theory to assumptions regarding bidders’ risk references and the outcomes of our experiment demonstrate that risk aversion cannot account for overly aggressive bidding in common value auctions. Rather, since the implications of Nash equilibrium bidding theory apply to a far wider variety of circumstances than in first-price auctions, there is far less scope to rationalize the failure of the theory under second-price rules. In addition, the failure of the comparative static implications of the model with respect to increasing numbers of bidders provides rather clear evidence that bidders fail to account for the adverse selection forces inherent in common value auctions, which failure is at the heart of the winner’s curse. In this context, the positive average profits observed in auctions with 4 or 5 bidders may be attributed to the reduced adverse selection forces associated with reduced N and to survivorship principles – experience with losses teaches bidders to bid less in these auctions without necessarily understanding the adverse selection forces at work.

The structure of the paper is as follows. In section II we describe our experimental procedures. Section III characterizes the Nash equilibrium bidding strategy
and offers a naive bidding model with its contrasting implications. The results of the experiments are reported in section IV. The concluding section summarizes our results.

2 Structure of the Auctions

Basic Auction Structure

Subjects were recruited for two-hour sessions consisting of a series of auction periods. In each period a single item was sold to the high bidder at a price equal to the second highest bid, with bidders submitting sealed bids for the item (a second-price, sealed bid procedure). The high bidder earned a profit equal to the value of the item less the second highest bid; other bidders earned zero profit for that auction period.

In each auction period the value of the item, \( x_0 \), was drawn randomly from a uniform distribution of the interval \([x, \bar{x}] = [$25, $225]\). Subjects submitted bids without knowing the realization \( x_0 \). Private information signals, \( x_i \), were distributed prior to bidding. The \( x_i \) were randomly drawn from a uniform distribution with upper bound \( x_0 + \varepsilon \) and lower bound \( x_0 - \varepsilon \). As such the \( x_i \) constitute unbiased estimates of the value of \( x_0 \) (or could be used with the end point values \( x, \bar{x} \) to compute unbiased estimates). Along with \( x_i \), each bidder received an upper and a lower bound on the value of \( x_0 \); these were \( \min\{x_i + \varepsilon, \bar{x}\} \) and \( \max\{x_i - \varepsilon, x\} \), respectively. The distribution underlying the signal values, the value of \( \varepsilon \), and the interval \([x, \bar{x}]\) were common knowledge. The value of \( \varepsilon \) varied across auctions (see Table 1); all changes in \( \varepsilon \) were announced and posted.

After all bids were collected, they were posted on the blackboard in descending order next to the corresponding signal values, \( x_0 \) was announced, the high bidder’s profit was announced (but not his/her identity), and balances were updated. Bids were restricted to be non-negative and rounded to the nearest penny.

Given the information structure and uncertainty inherent in common value auctions, negative profits would occasionally be realized even if the market were immediately to lock into the Nash equilibrium outcome. To cover this possibility, and to impose clear opportunity costs on overly aggressive bidding, subjects were given starting balances of $10.00. Profits and losses were added to this balance. If a subject’s balance went negative he was no longer permitted to bid and was paid his $4.00 participation fee. Auction survivors were paid their end-of-experiment balances and participation fees in cash.

Auctions with Public Information

After several auction periods we introduced bidding in two separate auction markets during each period. Bidding in the first auction market continued as before (called
private information conditions). After these bids were collected, but before they were posted, we publicly announced the lowest of the private information signals distributed, $x_L$, and asked subjects to bid again. (Subjects retained their original private information signals; no new private information signals were distributed.)

Profits were paid (or losses incurred) in only one of the two auction markets, determined on the basis of a coin flip after all bids were collected. Subjects were told that they were under no obligation to submit the same or different bids in the two markets, but should bid in a way they thought would “maximize profits.” All bids from both markets were posted along with the corresponding private information signals.

Observing the same subjects bidding in dual markets directly controls for between-subject variability and extraneous variability resulting from fluctuations in item valuations and private information signals. The dual market procedure creates two strategic choices which are theoretically separate, and subjects treat them as separate in a number of bidding contexts.3

Subject Experience and Varying Numbers of Bidders

The sessions are numbered in chronological order as they involved a common core of subjects, 17 in all, recruited in varying combinations (no two sessions involved exactly the same set of subjects and group composition varied unpredictable between sessions). All were MBA students or senior economics majors at the University of Houston. Each subject in session 1–3 had participated in at least one earlier session of second-price common-value auctions; 5 of 9 subjects in session 3 took part in session 1 or 2.4 Session 4 recruited back from this same subject pool, while recruiting two new subjects with experience in a parallel series of first-price common-value auction experiments. All of the subjects in sessions 5 and 6 had participated in one or more of sessions 1–4.

This sequence was designed to study performance in 7-bidder auctions during sessions 2, 3 and 4. Sessions 5 and 6 were designed to have 20 auctions with 4 active bidders, rotating participation among 7 subjects, and then to follow with 7-bidder auctions. Recruitment imperfections and bankruptcies created a slightly different pattern. Only 6 bidders survived auction series 2 and 4, while increasing the number of

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3 The Nash equilibrium model used to provide a game-theoretic prediction in a single unit auction presumes a von Neumann-Morganstern expected utility function. The independence axiom thus assumed, and the coin flip, make the optimal strategy in either market unaffected by the other. Battalio, Kogut and Meyer (1990), Kagel and Levin (1986) and Kagel, Harstad, and Levin (1987) report no systematic behavioral differences under dual market as compared to single market procedures.

4 We do not report outcomes of 5 sessions involving a total of 29 inexperienced subjects. The inexperienced-subject sessions followed procedures of section II(a) above, one beginning with 5 bidders, the others with 6. Any subject acquiring experience in those sessions (arbitrarily defined as having completed at least 15 auction periods without going bankrupt), was eligible to be recruited for the sessions reported here. Once recruited, subjects were invited back for later sessions without regard to earlier performance.
### Table 1. Experimental treatment conditions

<table>
<thead>
<tr>
<th>Session (number starting session)</th>
<th>Market Period</th>
<th>( \bar{\epsilon} )</th>
<th>Periods with Public Information (market periods)</th>
<th>Number of Active Bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(5)</td>
<td>1–18</td>
<td>$12</td>
<td>10–25</td>
<td>5(1–25)</td>
</tr>
<tr>
<td></td>
<td>19–25</td>
<td>$24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(7)</td>
<td>1–13</td>
<td>$12</td>
<td>9–20</td>
<td>7(1–10)</td>
</tr>
<tr>
<td></td>
<td>14–20</td>
<td>$18</td>
<td></td>
<td>6(11–20)</td>
</tr>
<tr>
<td>3(9)</td>
<td>1–8</td>
<td>$12</td>
<td>9–23</td>
<td>7(1–23)</td>
</tr>
<tr>
<td></td>
<td>9–14</td>
<td>$18</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>15–23</td>
<td>$30</td>
<td></td>
<td></td>
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<tr>
<td>4(7)</td>
<td>1–7</td>
<td>$12</td>
<td>12–25</td>
<td>7(1–21)</td>
</tr>
<tr>
<td></td>
<td>8–15; 21–25</td>
<td>$18</td>
<td></td>
<td>6(22–25)</td>
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<tr>
<td></td>
<td>16–20</td>
<td>$30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5(6)</td>
<td>1–5</td>
<td>$12</td>
<td>9–32</td>
<td>4(1–20)</td>
</tr>
<tr>
<td></td>
<td>6–14; 27–32</td>
<td>$18</td>
<td></td>
<td>6(21–26)</td>
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<tr>
<td></td>
<td>15–26</td>
<td>$30</td>
<td></td>
<td>5(27–32)</td>
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<tr>
<td></td>
<td>6–14; 25–29</td>
<td>$18</td>
<td></td>
<td>5(21–27)</td>
</tr>
</tbody>
</table>

Bidders led to a bankruptcy in both sessions 5 and 6. Below, we refer to 4-bidder auctions in these last two sessions as sessions 5A and 6A, and to auctions with more bidders as 5B and 6B.

### 3 Theoretical Considerations

The bulk of realizations of private information signals lie in the interval

\[
x + \varepsilon < x_i < \bar{x} - \varepsilon.
\]

Any signal satisfying (1) is unbiased:

\[
E[x_0 | X_i = x] = x.
\]

We restrict our theoretical analysis in the text to the realizations satisfying (1). \( ^5 \)

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\( ^5 \) The \( x_0 \) values and the associated signal values were all determined randomly, strictly according to the process described to the subjects. As such a sizable number, but small fraction, of signal values lie outside the interval (1). In cases where we can extend the theoretical analysis to cover such observations (for example, equilibrium bidding; see footnote 6) these data are included in the analysis along with observations in the interval (1).
In what follows we establish two benchmark models against which to evaluate the experimental results. A naive bidding model and the risk neutral symmetric Nash equilibrium bidding model (RNSNE for short). The naive bidding model represents an extreme form of naivete, while the RNSNE bidding model represents a very demanding form of rational play. We formulate these contrasting models as benchmarks against which to evaluate the outcomes reported and not because we believe that either one will exactly characterize behavior. We put much greater weight on the contrasting directional implications of the two models in response to increasing numbers of rivals and to the release of public information. Responses to these comparative static tests of the theory provide, in our opinion, a clearer test of whether or not subjects are sensitive to the contrasting item valuation forces underlying the two formulations.

Naive Bidding Under Private Information Conditions: A Model of the Winner’s Curse

The notion of a winner’s curse expressed in the petroleum engineering literature (Capen, Clapp and Campbell, 1971, Lohrenz and Dougherty, 1983) differs from game-theoretic models by a failure when formulating bids to fully account for the information implied by the event of winning. In what follows we specify an extreme version of this judgmental failure: bidders take no account of any information implied by winning, using only the private information signal \( x_i \) to estimate \( x_0 \), as in (2). A risk neutral bidder is willing to pay up to the expected value of the item, so that given the second-price bid rule, he has a dominant strategy of bidding (2) (Vickrey, 1961). In other words, naive bidders act as if they are in a private value auction, only they are bidding on a lottery with expected value (2). With all bidders using (2), expected profit, conditional on winning, is \( \varepsilon(3 - N\gamma)(N + 1) \) which is negative whenever \( N > 3 \). Note, we do not advocate (2) as a sensible bidding strategy, but are simply trying to develop implications of strategies using the naive expectation (2).

With risk aversion, naive bidders who are expected utility maximizers are only willing to pay up to the certainty equivalent of the lottery. As such a risk averse bidder employing (2) would bid below his signal value. Nevertheless, bids are likely to still be well above the Nash equilibrium bid function (equation 4 below).

Naive bidders do not react to changes in the number of bidders since the expectation, (2), is independent of the number of bidders. That is, this extreme form of the winner’s curse, because it takes no account of the adverse selection problem inherent in winning the auction, predicts an individual bid function that is not sensitive to the number of bidders. Further, since it is a dominant strategy for risk averse naive bidders to bid the certainty equivalent of the item, this insensitivity to numbers of bidders applies irrespective of risk preferences or asymmetry in these preferences.
Nash Equilibrium Bidding Under Private Information Conditions

Game-theoretic papers on second-price common-value auctions outline a model and proceed to characterize the symmetric Nash equilibrium, symmetric in both risk tolerances and strategy choices. Matthews (1977) and Milgrom and Weber (1982) showed that the function \( b(x) \) defined by

\[
E[U(x_0 - b(x)) | X_i = Y_i = x] = 0
\]

where \( x_i \) is the signal of bidder \( i \) and \( y_i \) is the highest signal among \( N - 1 \) rival bidders, is a symmetric Nash equilibrium. Levin and Harstad (1986) showed that this function is the unique symmetric Nash equilibrium. Further, following a result in Bikhchandani and Riley (1991), as extended in Harstad (1991), if there are more than 3 bidders, the symmetric equilibrium is the only locally nondegenerate risk neutral Nash equilibrium in increasing bid strategies.

Under risk neutrality the bid function satisfying (3) is

\[
E[X_0 | X_i = Y_i = x] = x - \varepsilon(N - 2)/N.
\]

This bid function discounts bids well below signal values, unlike the naive bidding model. Expected profit when all bidders employ (4) is \( 2\varepsilon(N - 1)/[N(N + 1)] \), which is positive, in marked contrast to the naive bidding model. As in the naive bidding model, the RNSNE predicts that the high signal holder always wins the auction. This follows directly from the assumption that all bidders have identical bid functions, the only difference being their private information, \( x_i \), regarding the value of the item.

Risk averse symmetric Nash equilibrium bidders will be below (4) resulting in even larger profits than under risk neutrality. This effect of risk aversion follows from the fact that the equilibrium bid function (3) is a simple expectation. This can be demonstrated through a straightforward application of the proofs in Pratt (1964, pp 128–129) to the symmetric bid function in (3).

The symmetric Nash equilibrium bidding model has the important comparative static prediction that individual bids must decrease with more rivals. This holds for all symmetric Nash equilibrium bidding formulations in second-price auctions irrespective of the form of the utility function. This follows from the fact that bidding

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6 An equilibrium is locally nondegenerate when the probability of any given bidder winning the auction is positive for all bidders.

7 For \( x_i < \bar{x} - \varepsilon \), the RNSNE bid function is \( \bar{x} + (x_i - (\bar{x} - \varepsilon))/N \). For \( x_i > \bar{x} - \varepsilon \), the RNSNE bid function is \( x_i - \varepsilon(N - 2)/N - (\bar{x} + \varepsilon - x_i)(N - 1)\theta(x)^{N-1}/[(1 - \theta(x)^{N-1})N] \) where \( \theta(x) = (x_i + \varepsilon - \bar{x})/2\varepsilon \).
in second-price auctions is based on a conditional expected value calculation, such as (3), which decreases with more bidders, independent of risk attitudes.\(^8\)

Harstad (1991, Proposition 2) extends the comparative static implications of increased numbers of rivals to asymmetric bidding models when the source of asymmetry is differences in bidders' risk preferences. That is, for a given set of asymmetric equilibrium bidding strategies that results in strictly increasing, locally nondegenerate bid profiles, reductions in the number of bidders results in increased bidding, provided that any given rivals' bid is equally likely not to be submitted; i.e., the identity of the bidders who have dropped out of the auction is not known prior to bidding (bidder identity reveals the risk profile of those who have dropped out of the bidding, which is likely to affect the remaining bidders' strategies). This proposition even extends to auctions in which the strategy profile is not an equilibrium. That is, as long as bidding strategies involve strictly increasing, nondegenerate bid profiles, a best response to an equally likely reduction in the number of bidders involves an increase in one's own bid, whether or not these bidding strategies satisfy the more demanding requirements of a Nash equilibrium.\(^9\)

### Naive Bidding Under Public Information Conditions

The winner's curse results from systematic overestimation of \(x_0\) by the high bidder as he fails to account for the adverse selection problem inherent in winning the auction. Under these conditions, the release of public information, particularly information that substantially reduces uncertainty about \(x_0\), is likely to result in a downward revision in the expected value of the item for a high signal holder, resulting in a lower bid and reduced average revenue for sellers.

To continue modeling an extreme case, suppose bidder \(i\) takes proper (Bayesian) account of the announced lowest signal, \(x_L\), but makes no inferences about others' signals compared to his, conditional on the event of winning. His estimate of the value of the item becomes

\[
E[x_0 | X_i = x, X_L = x_L] = x_L + \frac{N - 2}{N} \varepsilon + \frac{\theta^{N-1}}{1 - \theta^{N-1}} \frac{N - 1}{N} (x_L + 2 \varepsilon - x)
\]

\(^8\) More formally, this comparative static prediction is an unstated consequence of Theorem 5 in Milgrom and Weber (1982) applied to the symmetric bid function (3). Since (3) depends only on monotonicity of the utility function and not on risk attitudes (Levin and Harstad, 1986), the result applies to both risk averse and risk loving bidders. No comparable general result holds for first-price auctions; for the distributional parameters employed here and in Kagel and Levin (1986), the risk neutral first-price Nash equilibrium bid function falls with increased numbers of bidders, but this comparative static property is not invariant to risk attitudes.

\(^9\) A caveat is in order here. We have not been able to prove that there is a unique asymmetric equilibrium as a consequence of asymmetric risk preferences. Consequently there may be multiple asymmetric equilibria (we have not been able to demonstrate that such equilibria exist either). Harstad's proposition does not apply across two such increasing, locally nondegenerate asymmetric equilibria.
where $0 < \theta = \frac{x - x_L}{2\epsilon} \leq 1$. Under second-price rules, risk-neutral bidders using this estimate would adopt (5) as their bid function (using the same dominant strategy argument developed earlier).

Figure 1A illustrates the impact of announcing $x_L$ on individual bids, as predicted by the naive bidding model. The bid function, under private information conditions, which takes no account of rivals’ lower signals, (2), and the corresponding function under public information conditions, (5), calls for any bidder holding a signal above the value labeled $C_w(x_L)$ to reduce his bid upon learning $x_L$. On average, $C_w(x_L)$ occurs just below $x_0$, so that given the distribution of signal values, the naive bidding model predicts that the second-highest bidder will reduce his bid with $x_L$ announced most of the time (over 82% frequency for $N = 4$, over 97% for $N = 7$). Further, if all bidders employed (5), the dramatic reduction in bidding resulting from public information would restore a slight expected profit in auctions with 4 active bidders ($\epsilon/25$), but would still result in (modest) expected losses for auctions with 7 active bidders.

For market outcomes the implications of the symmetric, risk neutral, naive bidding model are clear. Making $x_L$ common knowledge will reduce expected revenue (increase bidders’ expected profit). As will be shown, this is in marked contrast to the prediction of the Nash equilibrium bidding model.

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10 This equation assumes $x$ strictly exceeds $x_L$, so that public information reveals a rival’s signal. The expectation is discontinuously higher for the lowest signal holder.

11 Frequency calculations for the location of $X_i = x_2$ here, and elsewhere in the text, are from a numerical simulation using 1000 sets of signal values for each value of $N$. 
Comparative Static Effects of Number of Bidders and Public Information on Behavior

Extensions of the naive bidding model to account for risk aversion yield more ambiguous results. Consider a naive bidder whose signal value is above \( x_0 \). Now, in addition to reducing the expected value of the item, there is reduced uncertainty about the value of the item. This reduced uncertainty will tend to result in bids that are closer to the expected value of the item. Thus, there are two opposing forces at work: the surprise value of \( x_L \) which promotes lower bidding and the reduction in uncertainty which promotes higher bidding. Assuming constant absolute risk aversion, which we argue for below, the net effect depends on the extent of bidders' risk aversion. As long as bidders are not too risk averse (a coefficient of constant absolute risk aversion of .10 or less) numerical calculations based on the expected location of the price setters signal show that the net effect of public information will be a lower bid (over the range of \( \varepsilon \) values employed). These two opposing forces are also at work in a naive bidding model in which bidders have asymmetric risk preferences.

Nash Equilibrium Bidding Under Public Information Conditions

Extensions of the common value auction model show that releasing public information, additional information that is affiliated with the variables of the model, will increase average revenue (reduce bidders' expected profits) under the RNSNE (Milgrom and Weber, 1982). Announcing the lowest of the private information signals drawn, \( x_L \), sharply reduces uncertainty concerning \( x_0 \), thereby markedly increasing average revenue. This occurs even though these announcements do not, on average, alter the expected value of the item for the high signal holder as

\[
E[x_0 \mid X_i = x_1] = E[E[x_0 \mid X_i = x_1, X_L] \mid X_i = x_1]
\]

However, with affiliation, the announcement of \( x_L \) increases the ex post expected value of the item for the second highest, and lower value, signal holders; they are surprised on average, and raise their bid. Since the second highest bid constitutes the sale price, average revenue increases.

In the RNSNE each bidder acts as if his signal value is higher than his rivals'. Under the uniform distribution, this assumption and the announcement of the lowest signal provide a bidder with a sufficient statistic for the aggregate information possessed by all bidders: the average of \( x_i \) and \( x_L \). Bidding \( (x_i + x_L)/2 \) is the dominance solvable outcome of the game.\(^{12}\)

Under the dominance solvable solution, announcing \( x_L \) nearly halves bidders' expected profit to \( \varepsilon/(N + 1) \). Further, any bidder for whom

\[
X_i = x < x_L + 2\varepsilon(N - 2)/N =: C(x_L)
\]

\(^{12}\) If a single outcome remains after finite iterations of deleting dominated strategies, the game is dominance solvable (Moulin [1979]). The argument here parallels Harstad and Levin [1985]: given the underlying distribution of signal values, with symmetry and risk neutrality any bidder is willing to bid at least as aggressively as \( (x_i + x_L)/2 \) independent of rivals' behavior, but no more aggressively than this against rivals bidding at least \( (x_i + x_L)/2 \).
will increase his bid following announcement of $x_L$. Remarkably, for our distributional parameters, this characterization does not depend on risk neutrality.

**Proposition:** For any concave utility function, any bidder $i$ with $X_i = x < C(x_L)$, $x$ in the interval (1), raises his bid with $x_L$ publicly announced, in the symmetric Nash equilibrium. (A proof is in the appendix.)

Figure 1B illustrates the impact of announcing $x_L$ on individual bids under the RNSNE. The bid function under private information conditions, (4), intersects the bid function with public information at $C(x_L)$, which lies beyond the expected location of the second-highest signal. The second-highest signal holder will find $x_i < C(x_L)$ over $\theta$% of the time. This results in substantially greater relative frequency of increased bidding on the part of the second highest bidder than in the naive bidding model. Furthermore, as the Proposition indicates, this frequency is not diminished by risk aversion in the symmetric Nash equilibrium.

Comparing Figures 1A and 1B we find important differences in predicted changes in bidding patterns for all individual bidders under the naive bidding model compared to the symmetric Nash formulation. Both models predict increasing bids, following the announcement of $x_L$, for signal holders in the interval $x_L < x_i < C(x_L)$. Further, for $x_i > C(x_L)$, the naive bidding model predicts reduced bidding, while the Nash equilibrium bidding model permits higher or lower bidding, depending upon bidders' risk attitudes. However, for signal values in the interval $C_w(x_L) < x_i < C(x_L)$, the naive bidding model predicts decreased bidding, while the Nash model requires increased bidding. This provides a basis for distinguishing between the two models at the level of individual bidding behavior.
Although as shown, in a symmetric equilibrium we can predict individual responses to public information by risk-averse bidders, it takes constant absolute risk aversion on bidders' part to insure that public information will raise average revenues in a second-price auction (Milgrom and Weber, 1982). On this basis, Milgrom and Weber (1982) note that reduced average revenue can only arise when the range of possible wealth outcomes from the auction is large (so that the bidders' coefficients of absolute risk aversion may vary substantially over this range). The range of possible wealth outcomes from our auctions is relatively small, and announcement of XL provides a marked reduction in uncertainty about the value of xo. As such we would anticipate that announcement of XL will result in increased average revenue (reduced bidders' profits) under a symmetric Nash equilibrium in our experiment.

As with the effects of changing numbers of bidders, the comparative static implications of increased revenue following the announcement of public information extends to asymmetric Nash equilibria as well. That is, in an asymmetric, strictly increasing, locally nondegenerate equilibrium, with constant absolute risk aversion, on average, the two highest bidders' willingness-to-pay is raised by the release of unbiased public information about the value of the item (Harstad, 1991, Proposition 1). In other words, release of public information raises expected revenue even with asymmetric constant absolute risk preferences.13

4 Experimental Results

Bidding Patterns with Private Information

Table 2 provides summary statistics of auction outcomes, where sessions are ordered by number of active bidders (shown in column 2).14 Columns 3 and 4 show the frequency with which the high signal holder won the auction, and the median rank-order correlation coefficient between bids and signals for each auction series. Data here are relevant to evaluating the extent to which an adverse selection problem existed, conditional on the event of winning the auction. Mean profit predicted under the naive bidding model, reported in column 5, serves to identify maximum expected losses, with bidders suffering from an extreme form of the winner's curse. Columns 6–8 compare actual profit earned and profit predicted under the RNSNE.

Looking at columns 3 and 4 in Table 2 we see that although the high signal holders fell well short of winning all auctions, as any symmetric bidding model pre-

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13 The caveat of footnote 9 holds here as well.
14 Our computations include all auctions under dual market procedures, whether or not bidders actually made profits (or losses) as a consequence of our coin flip rule. Since the coin flip was made after bids from both markets had been submitted, its outcome should not affect decisions. Dry run auctions with no money at stake, used at the beginning of a session to refresh bidders' memories, are not included in the analysis. Within a given series of auctions, there appears to be little systematic variation in bids over time, notwithstanding variations in N and e.
Table 2. Market outcomes under private information conditions

<table>
<thead>
<tr>
<th>Session (# auctions)</th>
<th># Active Bidders</th>
<th>Percentage of Auctions Won by High Signal Holder</th>
<th>Median Rank-Order Correlation Coefficient Between Bids and Signals</th>
<th>Average Profit (standard error mean)</th>
<th>Profit as a Percentage of RNSNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A (20)</td>
<td>4</td>
<td>70.0</td>
<td>.90</td>
<td>-3.49 (1.83) 3.34** (1.48) 5.49 (2.09)</td>
<td>60.8</td>
</tr>
<tr>
<td>6A (22)</td>
<td>4</td>
<td>90.9</td>
<td>.80</td>
<td>-2.71 (1.82) 5.42** (1.73) 6.68 (1.51)</td>
<td>81.1</td>
</tr>
<tr>
<td>6B (7)</td>
<td>5</td>
<td>42.9</td>
<td>.70</td>
<td>-5.86 (2.33) 3.24 (4.84) 7.61 (2.51)</td>
<td>42.6</td>
</tr>
<tr>
<td>1 (25)</td>
<td>5</td>
<td>40.0</td>
<td>.80</td>
<td>-5.69 (1.04) 1.11 (1.18) 3.45 (1.93)</td>
<td>32.2</td>
</tr>
<tr>
<td>5B (12)</td>
<td>5-6</td>
<td>58.3</td>
<td>.76</td>
<td>-9.03 (3.22) -5.84* (2.89) 6.19 (3.91)</td>
<td>-94.5</td>
</tr>
<tr>
<td>2 (20)</td>
<td>6-7</td>
<td>70.0</td>
<td>.84</td>
<td>-5.10 (1.01) -1.01 (1.28) 4.23 (1.14)</td>
<td>-23.9</td>
</tr>
<tr>
<td>4 (25)</td>
<td>6-7</td>
<td>44.0</td>
<td>.86</td>
<td>-10.80 (1.16) -.50 (1.19) 2.04 (1.95)</td>
<td>-24.5</td>
</tr>
<tr>
<td>3 (23)</td>
<td>7</td>
<td>43.5</td>
<td>.79</td>
<td>-9.44 (1.40) -3.06** (1.45) 4.72 (1.25)</td>
<td>-64.8</td>
</tr>
</tbody>
</table>

*: Significantly different from zero at 10% level in two-tailed t-test
**: Significantly different from zero at 5% level in two-tailed t-test

dicts, they won 57.8% of them, significantly more than would be expected on the basis of chance factors alone. Further, the median rank order correlation coefficient between bids and signals within each auction period consistently exceeded .70, again well beyond what would be expected on the basis of chance. These two results indicate that conditional on the event of winning, a bidder's signal was a biased estimate of the item's value. Further, the high positive rank order correlations between bids and signal values strongly supports the assumption of local nondegeneracy.

Profit reports in Table 2 show substantial differences conditional on the number of active bidders. In auction series with 4 or 5 active bidders, profits were consistently positive, averaging $2.78 per auction period overall, 52.8% of profits predicted under the RNSNE. Here profits are consistently closer to the predictions of the benchmark RNSNE model then the naive bidding model. In contrast, in auction series with 6 or 7 active bidders, average profits were consistently negative, averaging $-2.15 per

Note that a risk-averse equilibrium model would predict higher profits. The possibility that observed profit levels result from risk-loving may be discounted; subjects drawn from the same population for first-price, affiliated private value auctions exhibit no signs of risk loving in a setting free of item value estimation complexities (Kagel, Harstad, and Levin, 1987).
auction period, compared with predicted profits of $3.97 per period under the RNSNE. Here the naive bidding model predicts an average loss of $8.88, so bidders had made some adjustments to the adverse selection problem, but these were clearly inadequate to avoid losses (profits are almost equidistant between the two benchmark models). One obvious reason for the decrease in profits between small and large groups is that expected profits have decreased for the benchmark RNSNE. As such it is worthwhile pointing out that average profit decreased $4.93 per auction, substantially more than the decline predicted by the RNSNE, $1.29 per auction.

Table 3 shows the relationship between profits and $\beta$. We continue to distinguish between auctions with small (4–5) and large (6–7) numbers of bidders, and the few observations at $\beta = 24$ are pooled with $\beta = 30$ observations. In auctions involving 4–5 bidders, observed mean profit increases with $\epsilon$. However, with 6–7 active bidders, observed losses increase with increases in $\epsilon$, with the largest losses earned at $\beta = 24–30$. Although increasing $\epsilon$ increases profit opportunities in these auctions, it also involves increased opportunity for losses in cases where bidders fall prey to the winner’s curse.\(^{16}\)

Table 3. Profits and bidding under varying levels of $\epsilon$ (all profits in dollars)

<table>
<thead>
<tr>
<th>No. of Bidders</th>
<th>$\epsilon$</th>
<th>Average Profits (standard error mean)</th>
<th>Profits as a Percentage of the RNSNE Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Naive Bidding</td>
<td>Observed</td>
<td>RNSNE</td>
</tr>
<tr>
<td>4–5</td>
<td>12</td>
<td>-3.49</td>
<td>1.73*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.70)</td>
<td>(.92)</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>-2.07</td>
<td>3.09**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.05)</td>
<td>(1.39)</td>
</tr>
<tr>
<td></td>
<td>24/30</td>
<td>-8.49</td>
<td>3.66*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.01)</td>
<td>(2.15)</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>-5.15</td>
<td>-.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.56)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>6–7</td>
<td>18</td>
<td>-9.29</td>
<td>-1.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.22)</td>
<td>(-1.51)</td>
</tr>
<tr>
<td></td>
<td>24/30</td>
<td>-13.55</td>
<td>-4.53**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.20)</td>
<td>(-2.14)</td>
</tr>
</tbody>
</table>

*: Significantly different from zero at 10% level, two-tailed t-test.  
**: Significantly different from zero at 5% level, two-tailed t-test.

\(^{16}\) Limited-liability for losses sets up the possibility that players will bid more aggressively than under the one-shot Nash equilibrium as potential losses are truncated. However, initial cash balances were such that bidders were fully liable for all losses relative to the RNSNE prediction except with $N = 4$ and $\epsilon = 30$ (as the data indicates losses and gains do not vary systematically over time or with $\epsilon$ and the average cash balance of the 6 bidders who went bankrupt was $7.56 at the time of bankruptcy). Further, other things equal, limited-liability for losses is more of a problem with $N = 4$ than with $N = 6$ or 7, as the Nash model calls for reduced bidding with increased numbers of bidders (whereas the data shows positive profits with $N = 4$ and losses with $N = 6$ or 7, the exact opposite of the pattern implied by a limited-liability argument). See Hansen and Lott (1991), Lind and Plott (1991), and Kagel and Levin (1991) for further discussion of limited-liability issues in the context of first-price common value auctions.
Table 4 provides evidence on the effect of increasing numbers of bidders on individual bidding using fixed effect regression models (so that each subject has their own intercept term). The first specification is suggested by the RNSNE bid function (4), as the response to changing $\varepsilon$ is conditional on the number of active bidders. Specification 2 differs only by adding an intercept shift parameter for increasing numbers of bidders. Both fixed effects estimates restrict the subject dummy coefficients to sum to zero, so that the negative intercept values reported represent a small fixed markdown component to bids.

Under both specifications, the $x_i$ critically influence bids, with an estimated coefficient value of 1.0. Further, the coefficient on $\varepsilon$ is negative, and statistically significant, under both specifications. However, it is substantially smaller in absolute value than the RNSNE equilibrium prediction: with $N = 4$ and 5, the RNSNE model predicts coefficient values of $-0.50$ and $-0.60$, compared to the estimated value of $-0.33$, differences which are statistically significant as well. Finally, under both specifications there is essentially no response to increasing numbers of rivals as the coefficient values for these variables are all small in size, and nowhere close to being statistically significant at conventional levels. Thus, individual bidders generally fail to reduce their bids in the presence of the increased adverse selection problem resulting from more rivals, consistent with the naive bidding model.

Though the results in Table 4 are indicative of overall bidding patterns, the following two restrictive assumptions underlie the analysis: (1) we impose the same
bidding function on all subjects and (2) we employ questionable assumptions regarding independence of observations between auction periods. We address the first issue by running individual subject regressions. Recall that the Nash model prediction about lower bids with increased numbers of bidders extends to asymmetric bidding models (where the source of asymmetry is differences in bidders risk preferences) and to auctions in which the strategy profile is not an equilibrium. Hence, we should see each individual subject reducing their bids with increased numbers of rivals.

But this still leaves open the question of independence of observations between auction periods. Fitting individual subject bid functions (like those in Table 4) to the data within each experimental session typically shows an absence of serial correlation (as measured by the Durbin-Watson statistic). This is consistent with independence of observations within a given experimental session. However, fitting individual subject bid functions across experimental sessions and examining the residuals, we typically find experimental session effects. That is, the residuals all tend to be positive in one experimental session and negative in another, indicating experimental session effects. There are two equally plausible reasons for these effects: (i) subjects are responding to the bidding patterns of others in their group and/or (ii) there is learning across experimental sessions (for which we have strong evidence, at least in first-price common value auctions; see Garvin and Kagel, in press). Although we cannot sort out between these alternatives on this data set, introducing dummy variables (intercept shifts) into the regressions to capture these experimental session effects results in reasonably well behaved residuals.

After correction for these experimental session effects, we are left with 11 subjects for which we can identify $N$ effects using either the first or second bid functions specified in Table 4. (We fit model 2 to the data wherever we could. However, for about half the subjects the experimental session dummy variables meant that we could not identify the dummy variable $D$ in model 2. In these cases we fit model 1 to the data.) On the basis of these regressions we classified subjects as bidding higher, lower or the same across values of $N$. In determining whether or not subjects bid higher or lower (as opposed to the same) we employed a $\alpha$ level; i.e., in model 2 the F-statistic associated with the dummy variable $D$ and DEPS had to be significantly different from 1.0 at the .20 level or better, while in model 1 the t-statistic associated with the DEPS variable had to be significantly different from 0 at the .20 level (two-tailed t-test). Further, in cases where the signs of the $D$ and DEPS dummy variables differed (indicating that the effect of $N$ varied with $e$), we counted a subject as not changing their bid if there were $e$ values for which the net effect was zero (2 subjects fall into this category). Using these rules, we classify 2 subjects as bidding less with increased $N$ (as Nash equilibrium bidding theory requires), 3 subjects as bidding more.

17 Another possibility is “super game” effects as the repeated interactions might allow subjects to obtain outcomes that could not be achieved in a one-shot game. However, as noted in the concluding section of the paper these super game effects are not consistent with the profit levels observed.

18 The choice of significance level here is fairly arbitrary. Using the standard .05 or .10 level seems too severe since the theory makes no predictions regarding the variance in bidders’ responses to changes in $N$. However, using a more common significance level does little to alter the classifications; e.g., using a 5% significance level, we classify 2 subjects as bidding less with increased $N$, 2 subjects as bidding more, and 7 as no change.
with increased $N$ (in direct contradiction to the Nash model's predictions), and 6 as not changing their bids (in contradiction to the Nash model, but consistent with the naive bidding model). That is, only 2 of 11 subjects were bidding consistent with the Nash model's predictions.

Assuming that these directional responses to increases in $N$ are independent across subjects (a point we will return to in the concluding section of the paper), and that our subject population represents a random draw from the population as a whole, the proportion of the parent population we might expect to bid consistent with the Nash model's predictions (using a 95% confidence interval) may be as low as 3% or as high as 56%. In other words, we might expect as many as 97%, but no fewer than 44%, of the parent population to bid the same or to increase their bids as $N$ increases, which is inconsistent with the Nash model's predictions. We conclude that a substantial part of the parent population is unlikely to take account of the heightened adverse selection problem inherent in auctions with more bidders, which failure underlies the winner's curse. 19

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19 We have a good check on the robustness of these results. In session 5 (when subjects had maximum experience), we employed a cross-over design with subjects first bidding in games with $N = 4$ and then in games with $N = 6$ (observations are limited to $e = $ $30 since a bankruptcy reduced $N$ after several periods with $N = 6$). Limiting the analysis to signals in the interval (1) and computing the average discount factor $(x - b(x)$ where $b(x)$ is the bid associated with signal value $x$) for $N = 4$ versus $N = 6$ we obtain the following results:

<table>
<thead>
<tr>
<th>Subj.</th>
<th>Average $x - b(x)$</th>
<th>Average $x - b(x)$</th>
<th>$t$-statistic</th>
<th>degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 4$</td>
<td>$N = 6$</td>
<td>(prob $t = 0$)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.1</td>
<td>-12.7</td>
<td>1.76 (.13)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>15.7</td>
<td>23.7</td>
<td>-2.13 (.09)</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>-3.7</td>
<td>-2.2</td>
<td>-.86 (.42)</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>16.8</td>
<td>17.5</td>
<td>-.64 (.54)</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10.1</td>
<td>9.4</td>
<td>.62 (.56)</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>12.6</td>
<td>5.7</td>
<td>2.41 (.05)</td>
<td>7</td>
</tr>
</tbody>
</table>

$^a$ Difference between $x - b(x)$ with $N = 4$ less $N = 6$.

Using the .20 significance rule to identify a change in discount rates (as in the text), 1 subject (S2) decreases her bid going from $N = 4$ to 6 (has a higher discount with $N = 6$ than with $N = 4$), 2 subjects increase their bids (S1 & S6), and 3 do not change their bids as $N$ changes.
Comparative Static Effects of Number of Bidders and Public Information on Behavior

Effects of Public Information on Revenue

Table 5 shows the actual effects of announcing $x_L$ on revenue, and the predicted effects under the RNSNE. The ability of public information to raise revenue appears to be conditional on the number of active bidders in the auction, and by association, on the presence or absence of a strong winner's curse (losses) under private information conditions.Pooling data from auctions with 4 or 5 active bidders, $x_L$ raises revenue an average of $0.25 per auction period, which is not significantly different from zero ($t = .21$), and amounts to 16.1% of the RNSNE model's prediction.

In contrast, in auctions with 6 or 7 active bidders, announcing $x_L$ reduces revenue an average of $3.98 per auction period, compared to the Nash equilibrium prediction that revenue will increase by $1.81 per period. Further, a $t$-test shows the decrease in revenue here to be statistically significant ($t = -3.43$, significant at better than the .01 level).

There is no theoretical reason why increasing numbers of bidders should, by itself, result in $x_L$ lowering rather than raising revenue. However, increasing numbers of bidders does exacerbate the adverse selection problem resulting in negative average profits (a winner's curse) under private information conditions. As the naive bidding model suggests, under these circumstances the release of public information may help correct for the overly optimistic estimate of the item's value held by the high bidder. Individual bidders' responses to $x_L$ support this conclusion, and suggest that residual traces of the winner's curse were responsible for $x_L$ not raising revenue nearly as much as predicted under the RNSNE bidding model with $N = 4-5$.

Table 5. Effects of public information on revenue

<table>
<thead>
<tr>
<th>Session (# auctions)</th>
<th>No. of Active Bidders</th>
<th>Average Change in Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual (standard error)</td>
</tr>
<tr>
<td>5A (12)</td>
<td>4</td>
<td>-.05 (3.09)</td>
</tr>
<tr>
<td>6A (14)</td>
<td>4</td>
<td>-1.12 (3.02)</td>
</tr>
<tr>
<td>6B (7)</td>
<td>5</td>
<td>2.06 (3.37)</td>
</tr>
<tr>
<td>1 (16)</td>
<td>5</td>
<td>.35 (1.40)</td>
</tr>
<tr>
<td>5B (12)</td>
<td>5-6</td>
<td>-5.91 (3.61)</td>
</tr>
<tr>
<td>2 (12)</td>
<td>6-7</td>
<td>-2.37 (1.80)</td>
</tr>
<tr>
<td>4 (14)</td>
<td>6-7</td>
<td>-1.25 (1.52)</td>
</tr>
<tr>
<td>3 (15)</td>
<td>7</td>
<td>-4.02** (1.86)</td>
</tr>
</tbody>
</table>

*: Significantly different from zero at 5% level in two-tailed t-test
Table 6 reports individual bidders’ responses to the release of public information. Responses have been categorized as increasing, decreasing, or not changing with the announcement of $x_L$, separated into regions by distance between the bidder’s signal and the lowest signal. In region (i), $x_i < C_w(x_L)$, both the naive and the symmetric Nash equilibrium bidding models predict that individual bids will increase with announcement of $x_L$. In region (iii), $x_i > C(x_L)$, both the naive model and the RNSNE model predict that individual bids will decrease with announcement of $x_L$ (no general prediction can be made here under Nash allowing for risk aversion). However, in region (ii), $C_w(x_L) < x_i < C(x_L)$, the naive model calls for bids to decrease with announcement of $x_L$, while the symmetric Nash model calls for bids to increase (even with risk aversion).

For signals in region (i), bidders generally responded in the appropriate direction, with 89.5% of all bids increasing for $N = 4–5$ and 79% increasing for $N = 6–7$. For signals in region (iii), most bids decreased as expected; 80.4% of all bids for $N = 4–5$ and 81% of all bids for $N = 6–7$. Finally, for signals in region (ii), where the symmetric naive and Nash models offer different predictions, we find a higher frequency of bids decreasing then increasing: 45.8% decreasing with $N = 4–5$, compared to 40.7% increasing; 57% decreasing for $N = 6–7$, compared to 30.1% increasing.

The relatively high frequency with which individual bids decrease, or do not change, for signal values in category (ii) goes a long way towards explaining why public information did not raise revenue as much as predicted under the RNSNE bidding model with small numbers of rivals, and resulted in decreased revenue with larger numbers of bidders. The second highest signal occurs in region (i) less than 18% of the time for $N = 4$, dropping rapidly to less than 3% for $N = 7$; it occurs in region (iii) a little over 30% of the time (the frequency here is insensitive to $N$).

The mechanism for public information to raise revenue under the Nash formulation depends critically on the fact that the second highest signal holder increases his bid in region (ii) when $x_L$ is announced. But the frequency with which individual bids de-

<table>
<thead>
<tr>
<th>Number of Bidders</th>
<th>Private Information Signal Categories (total # signals in category)</th>
<th>Increase</th>
<th>Decrease</th>
<th>No Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-5</td>
<td>$(i) \ x_i \leq C_w(x_L)$</td>
<td>89.5</td>
<td>3.5</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>(114)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(ii) C_w(x_L) &lt; x_i &lt; C(x_L)$</td>
<td>40.7</td>
<td>45.8</td>
<td>13.6</td>
</tr>
<tr>
<td></td>
<td>(59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(iii) x_i \geq C(x_L)$</td>
<td>8.7</td>
<td>80.4</td>
<td>10.9</td>
</tr>
<tr>
<td></td>
<td>(46)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(i) \ x_i \leq C_w(x_L)$</td>
<td>79.0</td>
<td>10.9</td>
<td>-10.1</td>
</tr>
<tr>
<td></td>
<td>(138)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-7</td>
<td>$(ii) C_w(x_L) &lt; x_i &lt; C(x_L)$</td>
<td>30.1</td>
<td>57.0</td>
<td>12.9</td>
</tr>
<tr>
<td></td>
<td>(93)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(iii) x_i \geq C(x_L)$</td>
<td>16.7</td>
<td>81.0</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>(42)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Comparative Static Effects of Number of Bidders and Public Information on Behavior

crease, or remain constant, in this category indicates that this mechanism is simply not working well in these auctions, irrespective of the number of participants. Finally, the consistency of changes in individually bids with the naive model's predictions provides further evidence that overly aggressive bidding is rooted in the failure to recognize the adverse selection problem conditional on winning the auction.

Table 7 reports market outcomes under public information conditions. Mean actual profit was positive in 7 of the 8 auction series, averaging 47.6% of profit predicted under the RNSNE ($1.57 per period actual vs $3.30 predicted). Although profits varied considerably across auction periods relative to the RNSNE model's predictions, unlike private information conditions they did not differ substantially conditional on the number of bidders present: with $N = 4-5$ average profits were 51.7% of predicted ($1.93$ per period actual vs $3.73$ predicted), and with $N = 6-7$ profits were 41.2% of predicted ($1.15$ actual vs $2.79$ predicted). That is, unlike private information conditions, the actual reduction in profits is proportional to the predicted reduction.

Table 7. Market outcomes under public information conditions

<table>
<thead>
<tr>
<th>Session (# auctions)</th>
<th>Number of Bidders</th>
<th>Percentage of Auctions Won by High Signal Holder</th>
<th>Median Rank-Order Correlation Coefficient Between Bids and Signals</th>
<th>Average Profit (Naive Bidding, Observed, RNSNE)</th>
<th>Profit as a Percentage of RNSNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A (12)</td>
<td>4</td>
<td>50.0</td>
<td>.40</td>
<td>2.69, 4.88</td>
<td>55.1</td>
</tr>
<tr>
<td>6A (14)</td>
<td>4</td>
<td>28.6</td>
<td>.30</td>
<td>2.93, 6.41**, 5.87</td>
<td>109.2</td>
</tr>
<tr>
<td>6B (7)</td>
<td>5</td>
<td>14.3</td>
<td>-.20</td>
<td>1.64, 1.17, 5.38</td>
<td>21.7</td>
</tr>
<tr>
<td>1 (16)</td>
<td>5</td>
<td>25.0</td>
<td>.30</td>
<td>-1.85, 0.02, 1.39</td>
<td>1.4</td>
</tr>
<tr>
<td>5B (12)</td>
<td>5-6</td>
<td>16.7</td>
<td>.31</td>
<td>-0.68, 0.07, 3.62</td>
<td>1.9</td>
</tr>
<tr>
<td>2 (12)</td>
<td>6-7</td>
<td>25.0</td>
<td>.14</td>
<td>0.01, 1.54, 2.90</td>
<td>53.1</td>
</tr>
<tr>
<td>4 (14)</td>
<td>6-7</td>
<td>42.9</td>
<td>.36</td>
<td>-.92, 1.21, 1.61</td>
<td>75.2</td>
</tr>
<tr>
<td>3 (15)</td>
<td>7</td>
<td>40.0</td>
<td>.56</td>
<td>-1.34, -0.43, 2.33</td>
<td>-18.5</td>
</tr>
</tbody>
</table>

**: Significantly different from zero at 5% level in two-tailed test

20 Similar results are reported for individual subjects for signals in region (ii), where the symmetric and Nash models offer different predictions: With $N = 4$ or $5$, 56% of the subjects (5 out of 9) decrease their bids or do not change their bids 50% of the time or more following the release of public information (contrary to the symmetric Nash model's predictions). With $N = 6$ or $7$, 80% of the subjects (8 out of 10) do the same.
so that the strong winner's curse present with large numbers of bidders under private information conditions has been eliminated.

Finally, as columns 3 and 4 of Table 7 show, both the frequency with which the high signal holder won these auctions, and the median rank order correlation coefficient between bids and signals, have deteriorated substantially relative to private information conditions (recall Table 2). However, both are still greater then one would expect on the basis of chance factors alone, so that the adverse selection problem is still present, but with substantially reduced intensity. The reduced correlation between bids and signals under public information conditions can readily be accounted for by the fact that announcement of $x_L$ substantially reduced the diversity of expectations regarding $x_0$, relative to private information conditions. This permits any underlying differences in aggressiveness, risk attitudes, or information processing capacities, to play an increased role in auction outcomes.

5 Summary and Conclusions

We have reported a series of second-price common value auctions with experienced bidders. Auctions with 4 or 5 bidders exhibit substantial positive profits with outcomes closer to the benchmark RNSNE then the naive bidding model. In contrast, in auctions with 6 or 7 bidders, there is some adjustment to the adverse selection forces as profits exceed the naive bidding model's prediction, but subjects consistently earn negative profits, as they suffer from a winner's curse.

The comparative static tests of the Nash equilibrium bidding model, which are theoretically quite robust and more revealing of the underlying behavioral process in second-price auctions, indicate strong elements of a winner's curse. The individual bid function does not change with varying number of rivals, consistent with the naive bidding model, and in direct contradiction to the Nash equilibrium bidding formulation. This provides rather striking evidence that losses in auctions with 6 or 7 bidders do not result from simple miscalculations, but from a failure to appreciate the adverse selection forces inherent in common value auctions. Announcing public information results in a less-than-significant increase in average revenue for auctions with 4 or 5 bidders, but a large decrease in revenue for auctions with 6 or 7 bidders. Further, the pattern of individual bidders' responses to public information suggests a failure to account reliably for the information implied by the event of winning. Finally, bidders are sensitive to the changes in $e$, which is consistent with the Nash model and is inconsistent with risk neutral naive bidders. But this outcome is consistent with risk averse, naive bidders.

One potentially troublesome problem with our experimental design is the use of a single set of 17 subjects, in different combinations, in the different experi-

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21 Pooling data across auction series, chance factors alone would account for 18% of all auctions being won by the high private information signal holder, compared to the high signal holder winning 30% of all auctions. The difference is significant at better then the 1% level using a binomial test statistic.
tal sessions (11 out of 17 participated in more than one session). This experimental design was motivated by the desire to provide subjects with maximum experience with the task at hand and to compare the effects of bidding in auctions with different numbers of bidders. But the design does raise questions about the independence of observations both within and across experimental sessions. That is, aside from any individual subject learning effects that might be present, are we justified in treating each auction period within a given experimental session as an independent observation? If we cannot treat each auction period as independent can we treat each experimental session as an independent observation or does the fact that subjects often played in more than one game with some of the same people mean that we really only have one observation?

There are two potential reasons for lack of independence in a small group experiment of this sort. First, the repeated interactions of individuals within an experimental session creates the potential for super game effects in which players may achieve outcomes that are not equilibria in single shot games. In the context of our experiment, these additional equilibria would typically involve subjects achieving some sort of collusive outcome (tacit or otherwise) resulting in higher profits than the single-shot RNNE. As such we can safely rule out these supergame effects as subjects consistently earned negative profits in auctions with 6 or 7 bidders. The second possibility, which is very real and consistent with game theoretic reasoning, is that the outcomes of the game (whether or not they involve equilibrium behavior or best responses to others' actions) reflect the composition of the group within a given experimental session. From this perspective we would have to rule out independence of observations between auction periods within a given experimental session (i.e., seeing negative profits in 9 out of 12 auction periods in experimental session 5 with \( N = 5 \) or 6, as we did, is not the same as seeing negative average profits in 9 out of 12 different experimental sessions with different groups of subjects). Moreover, the experimental sessions are not completely independent since there was some overlap of participants between sessions. However, if one is willing to make the assumption that subjects have stable bid functions which are reflected by our regressions, we can treat the 11 subjects for whom we could identify \( N \) effects in their individual subject bid functions as independent draws from some parent population, as we evaluate these effects after controlling for individual session effects.\(^{22}\) And these data are most damaging to Nash bidding theory, indicating that between 44% and 97% of the parent population will not consistently decrease their bids when \( N \) increases, thereby committing the winner’s curse. Of course, it is still possible that the outcomes we observe are peculiar to the particular history of play our subjects experienced. So in this respect our results require replication.

The results here closely parallel those of earlier reports of bidding in first-price common value auctions (Kagel and Levin, 1986; Dyer, Kagel and Levin, 1989; Kagel et al, 1989). The distinguishing characteristics of the present study is that in conducting

\(^{22}\) As noted, examination of the residuals from individual subject regressions support this argument, after the adjustment for experimental session effects. Also, unlike some games (e.g., coordination games), in our auction individual subject’s best response to an increase in \( N \) does not depend on what others’ responses are to these increases, provided certain mild regularity conditions are satisfied.
it in a second-price auction institution the testable implications of Nash equilibrium bidding theory apply to a far wider variety of circumstances than in first-price auctions. As such there is far less scope to rationalize the failure of Nash equilibrium bidding theory under the second-price rules.

One noticeable difference between bidding in second and first-price common value auctions is the frequency with which high signal holders win the auctions: 70.3% of all auctions were won by the high signal holders in first-price auctions (Kagel and Levin, 1986) compared to 57.8% here. In comparison, in auctions with affiliated private values, 80.4% of all first-price auctions were won by the high value holder compared to 73.4% of second-price auctions (Kagel, Harstad and Levin, 1987). Thus, there is a reduction in the frequency with high signal holders win the auction in going from private to common value auctions as well as in going from first-price to second-price auctions. One possible explanation for the reduced frequency of winning in common value auctions is that the item valuation component makes these auctions substantially more complex than private value auctions. As such there is more scope for individual subject bidding errors to express themselves, or for any inherent variability in players’ ability to process information to affect bids. With respect to differences in the frequency with which high signal holders win under first and second-price rules, we conjecture that it is the increased flatness in bidders’ payoff functions that is partly responsible for the differences observed. With a flatter payoff function, there are reduced monetary costs to deviating from the optimal bidding strategy. As such, any inherent variability in bidders behavior is less constrained in second-price auctions as the relative cost of deviating from the optimal bidding strategy has been reduced.

Appendix

This appendix proves the proposition in the text. It starts by providing a few known results on symmetric second-price auctions and proceeds by proving a lemma used in the proof of the proposition.

Matthews (1977) and Milgrom and Weber (1982) showed that the function \( b(x) \) defined by \( E[u(x_0 - b(x)) \mid X_i = Y_i = x] = 0 \), where \( X_i \) is the signal of bidder \( i \) and \( Y_i \) is the highest signal among the \( N - 1 \) rival bidders, is a symmetric Nash equilibrium (SNE). Levin and Harstad (1986), showed that this function is the unique SNE. With public information in the form of \( x_L \), the lowest private signal received by the participating bidders, the unique SNE becomes \( B(x, x_L) \) defined by \( E[u(x_0 - B(x, x_L)) \mid X_i = Y_i = x, X_L = x_L] = 0 \).

Consider the distributions functions \( F(x_0 \mid X_i = Y_i = x) \) and \( G(x_0 \mid X_i = Y_i = x, x_L = x_L(x)) \) where \( x_L \) is restricted to the interval (1) (i.e. \( x + \varepsilon \leq x_L \leq x - \varepsilon \)) and where \( x_L(x) \) is defined by

\[
E[x_0 \mid X_i = Y_i = x] = E[x_0 \mid X_i = Y_i = x, X_L = x_L(x)].
\] (A.1)
In the interval (1) $E[x_0 \mid X_i = Y_i = x] = x - \epsilon[(N - \epsilon)/N]$ and $E[x_0 \mid X_i = Y_i = x, X_L = x_L(x)] = (x + x_L)/2$. Thus

$$x_L(x) = x - 2\epsilon(N - 2)/N \quad \text{and} \quad dx_L(x)/dx = 1.$$  \hspace{1cm} (A.2)

Let $C(x_L)$ be the inverse of $x_L(x)$ thus

$$C(x_L) = x_L + 2\epsilon(N - 2)/N.$$  \hspace{1cm} (A.3)

Lemma: $F[x_0 \mid X_i = Y_i = x]$ is a Mean Preserving Spread (MPS) of $G[x_0 \mid X_i = Y_i = x, X_L = x_L(x)]$.

Proof: The way we define $x_L(x)$ in (A.1) assures that $F$ and $G$ have the same mean in interval (1).

$$F[x_0 \mid X_i = Y_i = x] = \begin{cases} 0 & x_0 \leq x - \epsilon \\ 1 - [(x + \epsilon - x_0)/2\epsilon]^{N-1} & x - \epsilon < x_0 < x + \epsilon \\ 1 & x + \epsilon \leq x_0 \end{cases}$$

$$G[x_0 \mid X_i = Y_i = x, X_L = x_L(x)] = \begin{cases} 0 & x_0 \leq x - \epsilon \\ (x_0 + \epsilon - x)N/4\epsilon & x - \epsilon < x_0 < x - \epsilon(N - 4)/N \\ 1 & x - \epsilon(N - 4)/N \leq x_0 \end{cases}$$

Define $S(x_0 \mid x) = F[x_0 \mid X_i = Y_i = x] - G[x_0 \mid X_i = Y_i = x, X_L = x_L(x)]$.

$$S(x_0 \mid x) = \begin{cases} 0 & \text{if } x_0 \in D_1 =: [x + \epsilon, x - \epsilon] \\ 1 - [(x + \epsilon - x_0)/2\epsilon]^{N-1} - (x_0 + \epsilon - x)N/4\epsilon & \text{if } x_0 \in D_2 =: [x - \epsilon, x - \epsilon(N - 4)/N] \\ -[(x + \epsilon - x_0)/2\epsilon]^{N-1} & \text{if } x_0 \in D_3 =: [x - \epsilon(N - 4)/N, x + \epsilon] \\ 0 & \text{if } x_0 \in D_4 =: [x + \epsilon, x - \epsilon]. \end{cases}$$

It is easy to verify that $S$ is a continuous function in $x_0$ and that $\int dS(x_0 \mid x) = 0$ on $\text{supp}[x_0 \mid x]$. (This will establish that $F$ and $G$ are indeed mean preserving.) $S = 0$ if $x_0 \in D_1 \cup D_4$ and $S < 0$ if $x_0 \in D_3$. Thus to show that $S$ satisfies the Integral Condition for MPS, it is sufficient to show that there exist $\xi \in D_2$ such that $S > 0$ as $x_0 > \xi$, $x_0 \in D_2$. $S(x - \epsilon \mid x) = 0$, i.e. $S$ is zero on the left boundary of $D_2$ but strictly increases there since

$$\lim_{x_0 \to x - \epsilon} \partial S/\partial x_0 = (N - 2)/4\epsilon > 0.$$  \hspace{1cm} \text{if } x_0 \to x - \epsilon \geq x - \epsilon

$S$ is concave in $x_0$ on $D_2(\partial^2 S/\partial x_0^2 < 0)$ if $x_0 \in D_2$, and negative on the right boundary of $D_2$. $S(x - \epsilon(N - 4)/4 \mid x) = -[(N - 2)/N]^{N-1} < 0).$ These facts in conjunction with the continuity of $S$ establish our proof.
Proof of the Proposition in the Text: Let $X_L = x_L$ be the public information revealed. $0 = E[u(x_0 - b(C(x_L))) \mid X_i = Y_i = C(x_L)] < E[u(x_0 - b(C(x_L))) \mid X_i = Y_i = C(x_L), X_L = x_L]$. The equality follows the definition of $b(x)$, the inequality is due to the fact (established in the lemma) that the distribution $F$ is a MPS of $G$ and $u$ is a concave function. The inequality and the definition of $B(x, x_L)$ imply that $B(C(x_L), x_L) > b(C(x_L))$. If $x < C(x_L)$ then by (A:2) $x_L(x) < x_L$. Thus for such $x$ we have $B(x, x_L) > B(x, x_L(x)) > b(x)$ where the first inequality is due to affiliation which implies monotonicity of these bidding functions (see Milgrom and Weber, 1982), and the second inequality is due to the result established just above. We showed that $x < C(x_L)$ implies $B(x, x_L) > b(x)$.

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Received September, 1993
Revised version September, 1993