Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry

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The logic of revenue comparisons for different types of common-value auctions is substantially altered if the number of participants, rather than being fixed, responds endogenously to the expected profitability from participating. In a thoroughly symmetric model, a seller may prefer that competition be indirect: an auction procedure in which fewer participants are needed to drive the expected profitability from participating down to the level obtainable in other auctions in the economy can attain higher expected revenue if a sale is sufficiently likely. This insight allows a complete revenue ranking of standard auction procedures, with endogenous entry.

I. Introduction

Questions of revenue comparison across auction procedures dominate the theoretical and empirical literature on auctions (see McAfee and McMillan 1987*a*). The most studied theoretical model, the independent private-values model, exhibits revenue equivalence across a wide variety of auction procedures (Vickrey 1961; Milgrom and Weber 1982, pp. 1092–93). That model presumes that each bidder knows with certainty the auctioned asset's value to him, an unlikely description for auctions of durable or resalable goods or of assets of uncertain quality. For these, the common-value model may be more

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useful: it assumes that the asset has a common but unknown value, and bidders have private, imperfect information about this value. For common-value auctions (generalized), Milgrom and Weber (1982) provide expected revenue rankings across several standard auction procedures, as well as impacts on revenue of such seller choices as publicly announcing information correlated with the auctioned asset's value.

Revenue comparisons cited are based on symmetric Nash equilibrium outcomes for an *exogenously given* number of bidders. These analyses thus proceed as if no other auction (or other transaction) may offer a substitutable economic opportunity. An alternative approach, taken here, assumes that the number of competing bidders corresponds endogenously to expected profitability from competing. In symmetric equilibria of multistage games in which the last stage will be a common-value auction, with the number of bidders determined in a prior participation stage, inferences can be drawn that depend on the auction procedure only through the equilibrium number of participants. When fruition (i.e., a sale occurs) is sufficiently likely, an extension of the common-value revenue comparisons in Milgrom and Weber is obtained. With endogenous entry, however, a perhaps troublesome conclusion emerges: a seller often prefers an auction procedure because it generates *fewer* participants. Frequently voiced desires by sellers to generate larger numbers of bidders may need to be reevaluated.[1] (All results discussed here have a corresponding interpretation in procurement auctions.)

A few recent papers have adopted aspects of an implicit bidder participation decision, with an endogenous number of participants. In private-values auctions with entry, a seller's best reserve price is his own asset value (see Engelbrecht-Wiggans [1987] and Samuelson [in press] for examples). Milgrom (1981), for second-price auctions, and Matthews (1984) and Hausch (1988), for first-price auctions, consider special examples of common-value auctions. Extreme examples in Engelbrecht-Wiggans and Weber (1979) and Engelbrecht-Wiggans (1988) resist classification. Models with an uncertain (but exogenously determined) number of bidders appear in McAfee and McMillan (1987*b*), Matthews (1987), and Harstad, Kagel, and Levin (in press).

[1] This model assumes noncooperative bidders no matter what their number. A seller who believed that collusion could be avoided only by attracting many bidders may face a trade-off between lower chances of collusion and higher expected revenue given noncooperative behavior. A belief that collusion could be avoided either by many bidders or by a sealed-bid auction may help in part to explain the behavior of the U.S. Forest Service, which auctions timber harvesting rights, employing both English auctions and first-price auctions, with an apparent selection bias that may relate to the anticipated number of bidders (Hansen 1986).

II. Bidder Participation

Imagine a model in which potential bidders choose among a variety of auctions and other uncertain economic opportunities in which to invest their time and money. We shall examine a segment of the extensive form of such a model, relating to a particular auction. An indivisible asset with stochastic value V (to any bidder) is sold. A subset of potential bidders will participate, and a subset of participants will become actual bidders (the latter distinction is inessential unless an entry fee is imposed). Each participant i privately observes a signal X_i about V. The joint distribution of the affiliated variables $(V, \{X_i\})$ is common knowledge and is symmetric in the $\{X_i\}$.[2] For example, V could be uniform on $[L, H]$, with the X_i's independent and uniform on $[V - Q, V + Q]$, where Q is known. We shall refer to this example repeatedly, but simplifying it by considering only cases in which $L + Q \leq X_i \leq R - Q$.

The game segment unfolds as follows. First, the seller announces an auction procedure $F = (f, e_f, r_f) \in \mathcal{F} \times \mathbb{R}^2_+$, where f is an auction type (e.g., a first-price auction with none of the seller's private information revealed), $\mathcal{F}$ the set of auction types, e_f an entry fee, and r_f a reserve price. Second, each of a pool of potential bidders selects a probability of becoming a participant in this auction, based on F. Participation has two consequences: observing a signal, as mentioned above, and incurring a participation cost $c > 0$. This cost is likely to vary across auctions, but c is the same for all potential bidders in a given auction and is invariant to the procedure by which the auction is run. Unlike the entry fee, c does not accrue to the seller; it should be viewed as a forgone profitable opportunity (time consumed or inability to participate in another auction occurring elsewhere).[3] Third, each participant selects a probability of becoming an actual bidder, based on the signal observed and F. Each actual bidder pays the entry fee to the seller. Fourth, actual bidders select bidding strategies for the auction type f. The high bidder obtains the asset and pays the

[2] Any pair of affiliated variables satisfies the monotone likelihood ratio property, so a higher value for one makes a higher value for the other more likely. Independence conditional on V is a special case of affiliation (see Milgrom and Weber 1982, p. 1098 ff.).

[3] This consideration is missed if a view of substitute auctions is not at least implicitly present. Unwillingness of an additional potential bidder to participate need not imply zero net expected profit since participation costs may include expected profit opportunities forgone to participate in this auction (see Engelbrecht-Wiggans and Weber 1979). For example, suppose that a "wildcat" petroleum exploration firm has a limited quantity of trusted executives for pre-auction information gathering and processing and bidding strategy determination. Such a firm may view both a U.S. offshore sale and a North Sea offshore sale occurring in the same month as profitable opportunities; maximizing expected profit may imply focusing on one and forgoing the other.

seller a price determined by the auction type, the reserve price, and the bids submitted. Potential bidders are assumed to approach any auction with the objective of maximizing expected profit; that is, all are risk neutral.[4]

The solution concept applied is completely symmetric: each potential bidder selects the same probability of participating; in the event of participation, each selects the same function of the signal observed to determine the probability of actually bidding; each selects the same bidding strategy in the event he actually bids; and these selections constitute a Bayesian Nash equilibrium.

Under these conditions, a seller does not choose among auction procedures with the number of bidders invariant to his choice. Rather, the expected number of participants will vary with the auction procedure F so that expected profit equals participation cost. To formalize, let N be the number of potential bidders, $n(F)$ the equilibrium expected number of participants, and $a[F, n(F)]$ the equilibrium expected number of actual bidders. (Below, the dependence of a on n will be suppressed.) Then $n(F)/N$ is the equilibrium probability of participation. To simplify, $N > n(F)$, surely true for large enough N.

The asset being auctioned is worth V; the expected price paid by the winning bidder, in symmetric equilibrium given auction procedure F, a actual bidders, and $n \geq a$ participants, conditional on $V = v$, is represented as $p(F, a, n, v)$. As a convenient normalization, all prices—V, the X_i, e_f, r_f, and profit and revenue calculations below—are measured in units of participation cost, by setting $c = 1$. For our example, let $F = 2$ represent a second-price auction (without public information), with $e_f = r_f = 0$. When the uniform distributions mentioned are used, the symmetric equilibrium bid function is $b(x, n) = x - [Q(n - 2)/n]$. The high bidder pays a price equal to the second-highest bid; the expected value of the second-highest signal, given $V = v$, is $v + [Q(n - 3)/(n + 1)]$. So, for the example, $p(2, a, n, v) = v - [2Q(n - 1)/n(n + 1)]$.

The expected profit in the event of winning, gross profit in that c and e_f are not taken into account (but r_f is incorporated), is the expected difference between V and the conditional expected price:

$$w(F, a, n) = E\{V - E[p(F, a, n, \cdot)|V]\}. \tag{1}$$

For the example, $w(2, a, n)$ is simply $2Q(n - 1)/n(n + 1)$. The equilib-

[4] Opportunities to obtain similar assets in other auctions allow a potential bidder to diversify his portfolio of risky returns from participation in various auctions and other uncertain economic opportunities. Revenue comparisons for a particular seller are greatly simplified if a potential bidder's participation and bidding strategies are separable across auctions; separability results only if bidders' underlying (i.e., across-auction) utility functions exhibit risk neutrality or constant absolute risk aversion.

rium condition that the expected cost of participation equals the expected net profit is

$$1 + \frac{a(F)}{n(F)} e_f = \frac{w[F, a(F), n(F)]}{a(F)} \frac{a(F)}{n(F)}, \quad \forall F \in \mathcal{F} \times \mathbb{R}^2_+. \qquad (2)$$

For the example, (2) becomes $1 = 2Q(n - 1)/n^2(n + 1)$.

Notice which variables in equation (2) are determinable and what the source for the calculation is. The fundamental form of $a(\cdot)$ could be calculated as follows: for a given F and an arbitrary n, there would be a subset $\mathcal{X}$ of the support of X_i consisting of signals for which the expected profitability of winning justified paying the entry fee and bidding at least the reserve price. The probability that $X_i \in \mathcal{X}$ serves to determine the functional form of $a(F)$; the probability that a participant pays the entry fee is then symmetric: $a(F)/n(F)$. At the second stage, when potential bidders decide whether to participate, signals have not yet been observed, so the chances of winning are symmetric: $1/n(F)$ since the $a(F)$ terms in (2) cancel. The functional form of $w(\cdot)$ for given F and n is a common knowledge calculation, so (2) defines the functional form of $n(\cdot)$. Assume, as is natural, that $w[F, a(n), n]$ is decreasing in n for all $F \in \mathcal{F} \times \mathbb{R}^2_+$.

A remark on perspective: Only familiar assumptions in the auction literature are used here. (That changing the auction procedure cannot per se alter the asset's value is assumed by Milgrom and Weber [1982]. That changing the number of bidders does not alter the asset's common value is assumed by Milgrom [1981, sec. 5]. Both assumptions are made by French and McCormick [1984], Matthews [1984], and Hausch [1988]. One may expect a higher number of bidders to be drawn to the auction of a more valuable asset, but this is a ceteris paribus heuristic.) Moreover, some standard assumptions are more appropriate to the questions addressed here than to antecedent uses. Revenue comparisons of auction procedures most naturally arise *after* the seller has adopted any changes that enhance the asset's intrinsic value. Symmetric behavior seems the only sensible way to predict profitability of participation before the identity of participants is known. Finally, rationality is natural: it would be odd at best to investigate auction participation decisions that were based on the expected profitability attainable from the winner's curse or other irrational bidding behaviors (likely a better prescription would be, simply, not to participate).

III. Inferences for Revenue Comparisons

Expected revenue for auction procedure F, in symmetric equilibrium, is the sum of the expected price paid by the winning bidder and entry fees paid by all actual bidders:

$$\begin{aligned} R[F, a(F), n(F)] &= E\{p[F, a(F), n(F), \cdot]\} + a(F)e_f \\ &= E(E\{p[F, a(F), n(F), \cdot]|V\}) + a(F)e_f \\ &= E(E\{p[F, a(F), n(F), \cdot]|V\} - V + V) + a(F)e_f \\ &= E[V - n(F) - a(F)e_f] + a(F)e_f \\ &= E(V) - n(F), \end{aligned} \tag{3}$$

with substitution from (1) and (2). Thus the seller expects to receive asset value less aggregate participation costs.[5] The seller is of course also concerned with the likelihood of *fruition,* the event that at least one participant pays any entry fee and submits a bid no less than the reserve price. For N potential bidders, given participation probability π and probability α that a participant actually bids, the fruition probability is

$$\mathcal{P}(\alpha, \pi, N) = 1 - \sum_{i=0}^{N} (1 - \alpha)^i \binom{i}{N} \pi^i (1 - \pi)^{N-i}. \tag{4}$$

It is also convenient to let $\mathcal{S}(\alpha, \pi, N) = \alpha[1 - \mathcal{P}(\alpha, \pi, N - 1)]$, which is the chance that an assumed participant submits the only bid.

OBSERVATION 1. Any two auction procedures F and F' with the same equilibrium number of participants attain the *same* expected revenue (*a*) assuming fruition or (*b*) given zero reserve prices and entry fees.

Proof. Part *a* follows from (3). For part *b*, each participant bids, so the event of zero actual bidders has probability $\{1 - [n(f, 0, 0)/N]\}^N$ for both $(f, 0, 0)$ and $(f', 0, 0)$. Q.E.D.

Thus revenue comparisons of two auction procedures with equal numbers of bidders depend crucially on whether equal numbers is an assumption or a characterization. Equal numbers is solely an assumption for common-value auction revenue rankings in Milgrom and Weber (1982).

OBSERVATION 2. If auction procedure F' yields a participant higher expected profit than auction procedure F, when both are evaluated at the equilibrium number of participants for F, then F' will have a higher equilibrium number of participants than F.

[5] For the special cases they study, a corresponding result is discussed by French and McCormick (1984), is found by Hausch (1988), can be calculated in Milgrom (1981), and is found as an asymptotic approximation in Matthews (1984) (where the number of bidders is not necessarily an equilibrium level, but the participation costs are).

Proof. The proposition asserts $w[F', \cdot, n(F)] > w[F, \cdot, n(F)] \Rightarrow n(F') > n(F)$, which is a trivial consequence of (2). Q.E.D.

Observation 2 requires no particular relation between (e_f, r_f) and $(e_{f'}, r_{f'})$. Notice that any impact of entry fees on reducing the number of actual bidders may not be directly relevant; it is the number of participants the seller may wish to reduce.

OBSERVATION 3. Equilibrium expected revenue is *inversely* related to the equilibrium number of participants (*a*) given at least one actual bidder or (*b*) whenever fruition is sufficiently likely.[6]

Proof. Follows from (3). Q.E.D.

OBSERVATION 4. Let auction form F attract fewer participants than F', in equilibrium. A fruition probability sufficient for observation 3 to apply is

$$\mathcal{P}\left(\frac{a(F)}{n(F)}, \frac{n(F)}{N}, N\right) \geq [E(V) - n(F)]\mathcal{S}\left(\frac{a(F)}{n(F)}, \frac{n(F)}{N}, N\right). \quad (5)$$

Proof. Naturally, the ratio $a(F)/n(F)$ is nonincreasing in $n(F)$ (see Milgrom and Weber 1982, theorem 19), so it suffices to treat the case in which this ratio is a constant $\alpha \in (0, 1]$. For this case, the function $H(n) = [E(V) - n]\mathcal{P}(\alpha, n/N, N)$ extends equilibrium expected revenue from its domain of definition (taken to be $\{n \in \mathbb{R} | \exists F | n = n(F)\}$) to $\mathbb{R}$. For n^* defined by equality in (5), $dH/dn < 0$ on (n^*, N). Q.E.D.

Sufficiently likely fruition can often be obtained with few participants, particularly if c is large relative to $E(V)$, or if N is relatively small. For example, if $N = 20$, then for the example above with $[L, H] = [20, 80]$ and $Q = 17.15$, $(\mathscr{E}, 0, 0)$ ($\mathscr{E}$ an English auction) is an expected-revenue-maximizing auction, drawing 3.67 participants on average. However, with $Q = 11.78$, $(2, 0, 0)$, the second-price auction, draws 3.67 participants and higher expected revenue than the English auction. For $N = 6$, $[L, H] = [12, 48]$, and $Q = 11.38$, $(\mathscr{E}, 0, 0)$ draws an expected 2.91 participants, and the seller does not want more.

If a seller can switch to an auction procedure that will attract fewer participants, then each participant will have a higher chance of winning and so will in equilibrium settle for a lower expected profit in the event of winning. The winner's lower expected profit (as long as there

[6] Observations 1 and 2 can readily be extended to the "general symmetric model" of Milgrom and Weber (1982). The forces at work in observation 3 are also more general: in equilibrium with endogenous entry, given fruition, revenue will be nearer V the smaller the equilibrium number of participants. However, outside the common-value model, an additional consideration runs counter to this force: with more participants, V may be higher (in proportion to a first-order statistic).

is a winner) means that the seller receives an expected revenue nearer the asset's expected value.[7]

A natural comparative static question is whether a seller with a costless opportunity to reduce c gains by doing so. For given F, equilibrium $n(F)$ must rise as c falls. By (3), expected revenue will rise if $n(\cdot)$ responds to c inelastically. Such an inelastic response follows from (2) and $w(\cdot)$ declining with n when $e_f = 0$; when $e_f > 0$, the additional assumption that $a(\cdot, n)$ is increasing in n is sufficient.

The following revenue comparisons for common-value auctions with endogenous bidder participation can be inferred. Each assumes the condition in observation 4 and follows from the first three observations above and the theorems from Milgrom and Weber (1982) indicated in parentheses. (1) Expected revenue for an English auction is not less than that for a second-price auction (theorem 11). (2) Expected revenue for a second-price auction is not less than that for a first-price auction (theorem 15). (3) Publicly announcing any information the seller has that is affiliated with asset value cannot lower, and may raise, expected revenue for each of the three auction types discussed above (theorems 8, 9, 12, 13, 16, 17, and 18). (This does not say whether the seller should reveal or conceal the number of actual bidders.) (4) For each of the three auction types, higher entry fees and corresponding lower reserve prices generally raise expected revenue (theorem 19).[8]

Thus the seller's preference for an English auction, with public information announced, generalizes. To the extent that entry fees can reduce the number of participants without measurably reducing the likelihood of fruition, this will enhance expected revenue. With endogenous bidder participation, however, the reason for these prefer-

[7] Econometric studies relate higher revenue to a large number of bidders (McAfee and McMillan 1987*a*; Brannman, Klein, and Weiss 1987). A stated objective in the Outer Continental Shelf Lands Act amendments of 1978 (92 Stat. 629) is to increase the number of bidders in offshore mineral rights auctions. The U.S. Geological Survey conducts these auctions, many tracts at a time. When bids are in, the computer algorithm used to determine whether to award a "wildcat" tract does so automatically if the tract drew at least three bids. Some possible causes for this disparity are that (*a*) the econometric studies may have inexact proxies for asset value, (*b*) participants may not be employing risk-neutral symmetric equilibrium strategies (see Kagel and Levin 1986; Kagel, Levin, and Harstad 1988), (*c*) participation decisions may not be equilibrium choices (the equilibrium participation decisions posited here would not be best responses if rivals' bidding were not in equilibrium), and (*d*) this model may omit some key element of the markets studied.

[8] The precise statement is cumbersome. Suppose that (f, e_f, r_f) is regular ($z \in \mathscr{X}, z' > z \Rightarrow z' \in \mathscr{X}$) when evaluated at $n' = n(f, e'_f, r'_f)$, where $e'_f < e_f$, and r'_f has been set so that $a[(f, e_f, r_f), n] = a[(f, e'_f, r'_f), n]$ at both $n = n'$ and $n = n(f, e_f, r_f)$. (Milgrom and Weber refer to this as having the same screening level.) Then (f, e'_f, r'_f) is regular at n' but attains expected revenue that is at most that of (f, e_f, r_f).

ences is different: these policies *reduce* the equilibrium level of competition.

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