Theory and Methodology

On the role of discrete bid levels in oral auctions

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Abstract: Bids in oral auctions are restricted to discrete levels. This paper both examines the choice of levels at which bids will be allowed and also presents a simple model of the role of the discrete levels in bidding strategy. We consider two different distributions of bidders' values, identifying cases in which revenue is maximized by increasing intervals, by constant intervals, and by decreasing intervals. Moreover, conditions under which the choice of bid levels that maximizes bid taker revenue also maximizes economic efficiency are developed. We present a model of the economic trade-off between auction duration and step size. We consider the previously undiscussed issue of when economically motivated bidders should skip bid levels and when they should simply make minimum advances and develop a model in which it is equilibrium behavior always to make the minimum allowed advance.

Keywords: Bidding; Games; Economics; Modelling; Auction design

I. Introduction

Bids in sealed bidding and in oral auctions are restricted to discrete levels. In the extreme, the discrete levels are dictated by the finest division possible within the currency in use, e.g. pennies. However, at least in oral auctions, the auctioneer usually restricts the levels much more severely. Meanwhile, the standard theory of oral auctions assumes that the amount bid is a continuous variable. (See for example Milgrom and Weber (1982) and Harstad and Rothkopf (1991).) We are aware of only a single published paper, Yamey (1972), that deals at all with issues related to the role of discrete levels in oral auctions and none that deal with the impact of discrete levels on optimal bidding in oral auctions. (Chwe (1989) deals with discrete bid levels in first-price auctions as does Rothkopf (1969) briefly.) This paper strives both to examine the choice of levels at which bids will be allowed and to present a simple model of the role of the discrete levels in bidding strategy.

Cassady (1967) provides extensive descriptive material on oral auctions. In some oral auctions, the auctioneer will vary the minimum acceptable bid advance in response to the apparent level of competition. Often, in such auctions the auctioneer will accept smaller advances only if he or she is unable to obtain a
desired larger advance. In other auctions, the auctioneer and the bidders normally adhere to predetermined allowable bid levels.

Yamey (1972) considers art auctions of the latter kind (as conducted at Christie's and Sotheby's) and inquires into the logic of the use of widely spaced levels. (The levels at Christie's and Sotheby's tend to increase in steps of about 5%, even when such steps exceed £100,000.) His description makes it clear that tradition rather than economics plays a role. For example, at Christie's bids for silver and jewelry are in round numbers of pounds sterling but bids for paintings are in round numbers of guineas (1 guinea = £1.05). Furthermore, the step sizes, while approximately 5%, vary and may even decrease as bids increase (e.g. £5,000, £5,800, £6,000). Nevertheless, he puts forward an economic rationale for having large increments.

He notes that with the even spacing of bid levels expected revenue in the auction is given by

$$ E = v_2 - \frac{1}{2}D + pD(m - 1)/m, $$

where $v_2$ is the second highest valuation, $D$ is the bid increment, $p$ is the probability that the highest bid exceeds $v_2$ by $D$ or more, and $m$ is the expected number of bidders with valuations in excess of $v_2 - D$. If $D$ is small, $E$ will approximate $v_2$. For $E$ to be larger requires $p(m - 1)/m$ to exceed 0.5. Thus, Yamey argues that "if the object of the proportional-increment rule is to maximize sales proceeds, its adoption would seem to reflect the expectation or the experience that [bidders'] valuations typically or commonly form a pattern in which there is a significant gap between the two highest valuations, followed by a cluster of valuations; that this pattern occurs for objects of different levels of value; and that the spacing between valuations varies approximately pari passu with the level of the value of objects". However, Yamey does not consider the choice of levels for various value distributions. Nor is he concerned with more routine oral auctions such as those of fish, agricultural products, or repossessed cars in which auction duration is of economic significance in selecting bid increments.

This paper offers a small smorgasbord of analyses related to the impact of discrete bid levels in oral auctions. The intention is to open up several different approaches and not to present one complete unified theory.

Section 2 examines economic models of bid increments. We consider two different distributions of bidders' values, identifying cases in which revenue is maximized by increasing intervals, by constant intervals, and by decreasing intervals. Results are also developed giving conditions under which the choice of bid levels that maximizes bid-taker revenue also maximizes economic efficiency.

Section 3 presents a model of the economic trade-off between auction duration and step size. Section 4 raises the previously undiscussed issue of when economically motivated bidders should skip bid levels and when they should merely make minimum advances. We develop a model in which 'pedestrian bidding', i.e. always making minimum allowed advance, is optimal. Section 5 concludes the paper with a brief overview of our results.

The analysis below is described in the context of the independent-private-values model. Among the set of assumptions of that model, the crucial one for our results is statistical independence. So long as bidders' mechanisms to communicate privately with the auctioneer are secure (see Harstad and Rothkopf, 1991, particularly Section 3 and Theorem 4) and statistical independence of private information is maintained, value interdependence and associated residual uncertainty can be introduced into the analysis below with only expository complications.

We are acutely aware of the complicated nature of actual auctions. Our models are not put forward naively as realistic models of oral auctions, but in the hope that they may help in analyzing some of the issues involved in understanding particular auctions and provoking analyses of others.

2. Models of bid intervals

2.1. Preliminaries

First, consider a situation in which $n$ bidders independently draw values at random from some probability distribution on the interval $[a, b)$ and bid in an oral auction restricted to the bid levels $a (= l_0), l_1, l_2, \ldots, l_{m-1}, b (= l_m)$ which are num-
bered in increasing order. Like Yamey (1972), we assume throughout that no bidder will make a bid in excess of his or her value, that no sale takes place at a level \( l_k \) if any two bidders have a value greater than \( l_k \), that any bidder with a value in excess of level \( l_k \) is equally likely to be the bidder who bids that level, and that the levels immediately below and immediately above the second highest valuation are never jumped over in the bidding.

In order to calculate the bid-taker's expected revenue ('bid-taker' refers to the joint interest of the seller and the auctioneer) and other measures of effectiveness in this situation, it is useful to catalog three exhaustive and mutually exclusive ways that the winning bid can end up being \( l_t \):

Case 1. For \( i = 0, 1, 2, \ldots, m - 1 \), two or more bidders have values in the range \([l_i, l_{i+1})\) and no bidders have values \( v \geq l_{i+1} \).

Case 2. For \( i = 0, 1, 2, \ldots, m - 2 \), one bidder has a value \( v \geq l_i \), \( k \) bidders have values in the range \([l_{i-1}, l_i)\) and the bidder with the highest value is the one that makes the bid of \( l_i \), \( k = 1, 2, \ldots, n - 1 \).

Case 3. For \( i = 1, 2, \ldots, m - 1 \), one bidder has a value \( v \geq l_i \), \( k \) bidders have values in the range \([l_{i-1}, l_i)\) and the bidder with the highest value is not the one that makes the bid of \( l_{i-1} \), \( k = 1, 2, \ldots, n - 1 \).

Figure 1 illustrates these three cases.

2.2. Uniform distribution of values

If we assume that the probability distribution of values is the uniform distribution on \([0, 1)\), the

Illustrations of the Three Cases in which \( l_t \) Can Be the Winning Bid

<table>
<thead>
<tr>
<th>Key:</th>
<th>v indicates a bidder’s valuation</th>
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<tbody>
<tr>
<td></td>
<td>An arrow from a v to a bid level indicates the last bid made by the bidder</td>
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<td>with that valuation v</td>
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<tr>
<th>Allowable Bids:</th>
<th>( l_{i-1} )</th>
<th>( l_i )</th>
<th>( l_{i+1} )</th>
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<tr>
<td>Case 1:</td>
<td>v</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Two or more bidders (three shown) have values in the range ([l_i, l_{i+1})) and no bidders have values ( v \geq l_{i+1} ).</td>
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</table>

| Case 2:         | v            | v        | v            |
| One bidder has a value \( v \geq l_{i-1} \), one or more bidders have values \( v \) in the range \([l_{i-1}, l_i)\), and the bid at \( l_i \) is made by the bidder with the highest value. |

| Case 3:         | v            | v        | v            |
| One bidder has a value \( v \geq l_i \), one or more bidders have values \( v \) in the range \([l_{i-1}, l_i)\), and the bid at \( l_{i-1} \) is not made by the bidder with the highest value. |

Figure 1. Illustrations of the three cases in which \( l_t \) can be the winning bid
The first version of Case 1 gives the probability that all values lie below \( l_{i+1} \) times the probability conditioned on this restriction that all or all but one of the bidders do not have values below \( l_i \). In Case 2 and Case 3, \( \sum_{k=1}^{n-1} \frac{n}{k+1} (l_{i+1} - l_i) ^{k-1} \) is the number of combination of \( n - 1 \) objects taken \( k \) at a time which equals \((n - 1)!/(n - k - 1)!\).

We are interested in three related measures of effectiveness or ineffectiveness for this auction. One is the expected revenue received by the bid-taker, which we denote by \( E \). It is given by the sum from \( i = 1 \) to \( i = m - 1 \) of \( l_i \) times the sum of these three probabilities. The second, which we denote by \( L \), we call the expected 'lost revenue'. It is the amount by which on average the bid-taker revenue falls below the second highest valuation. This revenue is 'lost' because, in the continuous bid case, the equilibrium bid-taker revenue is the amount of the second highest valuation (Vickrey, 1961). Since the expected value of the second highest valuation is given by \((n - 1)/(n + 1)\),

\[
L = (n - 1)/(n + 1) - E.
\]

The third measure for the auction, which we will denote by \( I \), is the expected economic inefficiency of the auction, or more precisely the expected amount by which the value of the auction winner falls below the highest value. We now prove

**Proposition.** \( I = \frac{1}{2} L \).

Proof. We propose to show that no loss of economic efficiency occurs in Cases 2 and 3 defined above, while the expected loss of revenue in Case 2 is exactly offset by an expected gain in revenue in Case 3. Hence, the comparison turns on Case 1, and in this case the expected loss of revenue is twice the expected economic inefficiency. First, in Cases 2 and 3 the bidder with the highest valuation wins the auction. Hence, there is no economic inefficiency.

Second, in Case 2 the expected loss of revenue is \((l_{i+1} - l_i)(k/(k + 1))\) for \( i = 0, 1, 2, \ldots, m - 2 \), and \( k = 1, 2, \ldots, n - 1 \). In Case 3, the expected revenue gain is \((l_i - l_{i+1})/(k + 1)\) for \( i = 1, 2, \ldots, m - 1 \), and \( k = 1, 2, \ldots, n - 1 \). Hence, the expected inefficiency is given by the product of \( (k - 1)/k \), the probability that the highest value is not the winner, with the expected efficiency loss in this event, \((l_{i+1} - l_i)(k/(k + 1))\). This product is just half the expected lost revenue. \(\square\)

**Corollary 1.** Any change in bid levels that increases expected bid-taker revenue decreases expected economic inefficiency.

**Corollary 2.** The choice of bid levels that maximizes expected bid-taker revenue minimizes the expected economic inefficiency.

The proof of the proposition depends upon the assumed uniform distribution of values. While we have no rigorous proof to offer, note that as the interval between bids becomes small, the distribution within an interval, the critical item in the above proof, will tend toward uniform for any smooth distribution of values. (Given that the density is smooth and positive, the ratio of the supremum of the density in an interval to its infimum approaches one as the interval width approaches zero.) Therefore, we conjecture that for any such distribution the results in the proposition and its corollaries will be approached
asymptotically as the bid intervals are reduced in size.

One might hope that the results in the corollaries would hold independent of the distribution of values even though the theorem does not. However, this is not true either. The following example shows that Corollary 1 also depends upon the distribution of values even though the theorem does not. However, this is not true either. The following example shows that Corollary 1 also depends upon the distribution assumption: Consider two bidders drawing values from a distribution that has a 90% chance of being uniform on [0.9] and a 10% chance of being uniform on [w + 0.9, w + 1] for some width w. Assume that levels \( l_0 = 0 \), \( l_2 = w + 0.9 \), and \( l_3 = w + 1 \) have been chosen and that a value for \( l_1 \) is to be selected on (0, 0.9). The expected economic inefficiency is given by

\[
E = \frac{m^2 - 1}{3m^2} = \frac{1}{3} - \frac{1}{3m^2}.
\]

For arbitrary \( n \), but \( m = 2 \), we can obtain

\[
E = l_1 - l_1^n.
\]

The revenue maximizing value of \( l_1 \) is given by

\[
l_1 = (1/n)^{1/(n-1)}.
\]

Note that this is greater than \( \frac{1}{2} \) when \( n > 2 \). Hence, the revenue maximizing intervals will decrease with value in this case.

2.4. Exponential value distribution

Whether the optimal bid intervals increase, stay the same or decrease with magnitude depends not only upon the number of bidders, but also upon the distribution of values from which the bidders draw their values. In order to show this, we now turn to a situation in which two bidders draw values from an exponential distribution with density

\[
f(v) = \alpha e^{-\alpha v}, \quad 0 < v < \infty,
\]

and cumulative function

\[
F(v) = 1 - e^{-\alpha v}.
\]

We assume that bid levels 0 (\( \equiv l_0 \)), \( l_1 \), \( l_2 \), ..., \( l_{m-1}, \infty (\equiv l_m) \) are permitted and seek the values for them that maximize the bid-taker's expected revenue

\[
E = \sum_{i=0}^{m-1} l_i (e^{-\alpha l_i} - e^{-\alpha l_{i+1}})^2
\]

and the equations that result from setting the partial derivatives equal to 0 simplify to

\[
l_j = \frac{1}{2}(l_{j+1} + l_{j-1}), \quad j = 1, 2, \ldots, m - 1.
\]
the even spacing of \( m \), and a check firms that this spacing, the of \( m \) is given by.

Setting partial derivatives with respect to the bid levels equal to zero yields the equations

\[
l_j = \frac{l_{j-1} e^{-a_{i-1}} - l_{j+1} e^{-a_{j+1}}}{e^{-a_{i-1}} - e^{-a_{j+1}}} + \frac{1}{\alpha},
\]

\( j = 1, 2, \ldots, m - 2. \)

\( l_{m-1} = l_{m-2} + 1/\alpha. \)

These equations have the solution

\[
l_j = \sum_{i=1}^{j} X_i/\alpha,
\]

where \( X_{m-2} = 1 \), and successively smaller values of \( X_i \) can be found by solving numerically and iteratively the transcendental equation

\[
e^{-(X_i + X_{i+1})} = \frac{1 - X_j}{1 + X_{j+1}},
\]

\( j = m - 3, m - 4, \ldots, 1. \)

Numerical solution is made easier by taking advantage of the fact \( X_{j-1} \) can be expressed analytically as a function of the sum \( X_j + X_{j+1} \). The first few solutions in this decreasing sequence are \( X_{m-3} = 0.593, X_{m-4} = 0.424, X_{m-5} = 0.330, X_{m-6} = 0.270, X_{m-7} = 0.229, X_{m-8} = 0.198, X_{m-9} = 0.175, X_{m-10} = 0.156, X_{m-11} = 0.141, \) and \( X_{m-12} = 0.129. \) Note that the sequence of \( m \) terms \( X_1, X_2, \ldots, 0.424, 0.593, 1, \infty \) gives the relative widths of the revenue maximizing intervals when \( m \) levels are used and that these widths are increasing.

2.5. Rate of loss with equal bid intervals

Next, we return to the situation involving \( n \) bidders drawing values from the uniform distribution on \( [0, 1) \) and \( m \) divisions of the unit interval. Now, however, rather than looking for optimal divisions, we assume the equal spacing given by \( l_j = j/m, j = 0, 1, 2, \ldots, m \), that is optimal when \( n = 2. \) As noted above, if \( k \) of the bidders have valuations in the highest interval for which any bidders have valuations, then the expected lost revenue is \( (k - 1)/m(k + 1) \). The probability that all the bidders are in the top interval, i.e. \( k = n, \) is \( 1/m^n \) independently of which interval is the top one. For \( k = 1, 2, \ldots, n - 1 \), the probability that the \( i \)-th interval is the top one and has \( k \) valuations in it is given by \( C_{n,k}(i - 1)^{n-k}/m^n \). Hence,

\[
L = \frac{1}{m^n} \sum_{k=1}^{n} \sum_{i=0}^{m-1} C_{n,k} k-1^{n-k}/m^n.
\]

For \( n = 2 \) this gives, as is implied by our previous result,

\[
L = \frac{1}{3m^2}.
\]

For \( n = 3 \) it yields

\[
L = \frac{2}{3m^2} - \frac{1}{15m^3}.
\]

For higher values of \( n \), the final summation in the expression for \( L \) has \( n - 1 \) as its highest power of \( m \). Hence, \( L \) is asymptotically proportional to \( 1/m^2 \) for large \( m \).

3. Auction duration and optimal bid increments

Since revenue is maximized in the model of the previous section when the allowable interval between bids becomes small and since many oral auctions restrict significantly the interval between bids, we now turn our attention to a less detailed and more general model of an auction sale capable of explaining the use of significant intervals. In this model, there is a fixed lowest bid increment, \( D \), allowed. In the more precise context considered above, \( D \) can be thought of as \( 1/m \). We are interested in the choice of the size of the interval, in the context of costs associated with the duration of the auction. We assume that the time, \( T \), needed to sell an item is given by a fixed component, \( T_0 \), and a component inversely proportional to \( D \). Thus,

\[
T = T_0 + \kappa/D,
\]

where \( \kappa \) is the constant of proportionality. Based upon the asymptotic results above, we assume further that the expected lost revenue, \( L \), is well
approximated by $KD^2$ and that the expected economic inefficiency, $I$, is well approximated by $\frac{1}{2}KD^2$, where $K$ is the same constant of proportionality in both cases. If the cost of the auctioneer's time is given by $C_t$ and the auctioneer gets to keep a fraction $r$ of the sales revenue, then the myopic objective function of an auctioneer is to minimize

$$Z_a = rKD^2 + C_at(T_0 + \frac{\kappa}{D}).$$

The minimizing value of $D$, $D^*_a$, is $(\kappa C_at/2rK)^{1/3}$. Note that this is a cube root rule and, hence, $D^*_a$ and $Z_a(D^*_a)$ are quite insensitive to moderate sized errors in input.

If we denote the participation cost of all participants by $C_p$, then the overall economic optimum is achieved by minimizing

$$Z = \frac{1}{2}KD^2 + C_p(T_0 + \frac{\kappa}{D}).$$

The minimizing value of $D$, $D^*$, is

$$D^* = \left(\frac{\kappa C_p}{K}\right)^{1/3}.$$

The ratio of $D^*$ to $D^*_a$ is given by $(2rC_t/C_at)^{1/3}$. Unless the seller is the owner, $r$ will be much less than 1, and unless there are expensive auction facilities involved or the time of the assembled bidders is worth little, $C_t/C_at$ will be much larger than 1. While it is somewhat comforting that these distorting effects will tend to offset and that the cube root rule will dull the effect the remaining difference from unity, we believe that in many situations the choice of an interval by the seller will depend more on the auctioneer's desire to be fair and to attract bidders than on myopic short term profit optimization. This view is consistent with the sociological interpretation of auctions by Smith (1989).

4. On optimal dynamic behavior in English auctions

4.1. A decision-theoretic perspective

As discussed above in the Introduction, existing models of English auctions consider a bidder's strategy only in terms of limiting behavior — i.e., the maximum bid that the bidder should be willing to make. They ignore issues related to the discrete nature of allowable bids and the sequential nature of bidding. This section is concerned with the analysis of issues that arise when bidding is sequential and restricted to discrete values.

This section assumes that no significant information about the value a bidder should attach to the object being auctioned can be gleaned from the bidding of competitors. This assumption would be met in an independent private values context or when, as discussed in Harstad and Rothkopf (1991), competing bidders find it possible and worthwhile to disguise their intentions. A single auction with no side payments, collusion or other cheating is examined.

The analysis proceeds by examining a particularly simple situation and then generalizing the results obtained in it. We assume initially that allowable bid increments are equal and, to simplify matters, that the unit of money is chosen so that allowable bids correspond to the positive integers. We also assume that this unit of money is small enough that in the neighborhood of the final price bidders can be modeled as risk neutral with respect to risks involving no more than two units of money.

Consider the situation faced by a bidder whose reservation price for the object being auctioned, given his beliefs about the number of competitors and their information, is $J + \alpha$, where $J$ is an integer and $0 \leq \alpha < 1$. Let this bidder face a single competitor whose reservation price, $V$, (given the bidding so far) the bidder assesses to be drawn from a distribution starting at the present bid level. If the competitor makes a bid of $J - 1$, the bidder can do nothing better than make a final bid of $J$ and hope that it is successful.

If, however, the competitor makes a bid of $J - 2$, then the bidder must make a strategic choice of final bid between $J - 1$ and $J$. In this situation, bidding $J - 1$ risks foregoing a profit of $\alpha$ when the competitor's reservation price $V$ lies in the interval $[J, J + 1]$. However, bidding $J$ instead risks foregoing a profit of 1 when $V$ lies in the interval $[J - 2, J]$. If the conditional distribution of competitive value is the uniform distribution or the exponential distribution, the probability that $V$ is in the higher interval is less than the probability that it is in the lower one and, since $\alpha < 1$, the expected loss of jumping to $J$
must exceed the expected loss associated with a simple advance to \( J - 1 \).

Clearly, this result must hold not only for the uniform distribution and the exponential distribution of \( V \), but for any distribution with a density that declines or increases sufficiently slowly on the range \([J - 2, J + 1]\). The right-hand tails of most distributions, including the normal, meet this requirement. Note, however, that not all distributions do. In particular, a bidder who knows that a competitor can bid up to at least \( J \) has nothing to lose and, perhaps, something to gain by jumping the bid from \( J - 2 \) to \( J \).

The conclusions in this discussion need to be reconsidered, however, if the absolute interval between allowed bids increases as bids get higher as described in Yamey (1972). For example, if allowed bids are integers up to 10 and, above that, only multiples of 5 are allowed, a bidder with an evaluation of 14 would have an incentive to jump a competitive bid of 8 to a bid of 10 if he assessed his opponents conditional value distribution as flat or only slightly increasing in the range 8 to 15. However, in the Appendix we show that as long as the interval between allowed bids never increases by a factor of more than the square root of 2, a final advance from \( J - 2 \) to \( J - 1 \) always has at least as high an expected profit as a final jump to \( J \) if the conditional assessment of the best competitive valuation is flat or decreasing.

**4.2. A game-theoretic model**

Now consider a formal game in which two bidders each draw nonnegative reservation prices from commonly known nonincreasing distributions with nonincreasing densities (such as the exponential distribution or the uniform distribution on \([0, X]\)) and then bid against each other with positive integer bids in an English auction. In order to avoid somewhat artificial issues associated with the start of bidding, assume that the auctioneer starts the auction by selecting one of the bidders to be the winner at a price of 0 if the other does not bid.

Define 'pedestrian' bidding to be the strategy of advancing the competitor's bid by 1 until the auction is won or until such an advance will raise the bid above the bidder's reservation price. With this definition, pedestrian bidding by both bidders is an equilibrium.

To prove this, note first that, with any nonincreasing density, the conditional distribution of the variable given that the variable equals or exceeds any positive level has a nonincreasing density starting at that level. Hence, the logic above rules out final jumps by either bidder. Similar logic shows that against an opponent predictably using a pedestrian strategy earlier jumps are even less attractive.

If we now consider auctions with more than two bidders, the formal situation becomes messier. However, it is possible to shift the focus of a bidder's analysis to the highest reservation price among his competitors. If either the behavior of competitors or the size of the bidder's reservation price relative to the unit of advance make it sufficiently unlikely that there is more than one competitor who has a reservation price in the interval \([J, J + 1]\), then the analysis above makes a jump advance of the bid level unwise. However, if such a situation is sufficiently likely, a preemptive jump of the bid to 5 could conceivably be advantageous.

In real English auctions, bidders do upon occasion jump the bid. While the explanation for such behavior may conceivably lie within the realm for it delineated in the above analysis, there are at least two other relevant explanations. First, the bidder may be signalling in some way. For example, jumping the bid at an early stage of the bidding might be an attempt to influence the behavior of competitors either by indicating high interest in the object in order to induce pity, tacit collusion ("Don't force up the price on my favorite item and I won't do so on yours"), or disinterest in continuing to waste time bidding. (Cassady (1967) reports that in 1938, J. Paul Getty "departed from his usual practice and not only went to an auction but actually did his own bidding". He then quotes Getty from Getty and Le Vane (1953) as saying "The dealers, sensing my determination to secure certain objects had little interest in bidding prices up in vain, so resigned themselves to the inevitable"). Alternatively, a bidder jumping the bid might not be rational in the sense typically assumed in game theory, or might assume his rivals might not be. Any analysis of oral auctions containing skipped levels should consider these possibilities.
5. Conclusions

The role of discrete bid levels in oral auctions has received almost no analytic attention. This paper analyzes several issues related to it.

First, we have examined models of the allocation of a finite number of bid increments. We show that when bidders draw their private values from a uniform distribution, the bid-taker's expected loss of revenue due to the discreteness of bid levels equals twice the expected loss in economic efficiency due to the discreteness. In this case, given two bidders, spacing evenly any fixed number of allowed bid levels maximizes expected revenue and minimizes expected loss of economic efficiency. However, we present other examples in which either increasing spacing or decreasing spacing has these properties. Returning to the case of uniform value distributions and assuming equally spaced bid levels, we find that as the number of levels becomes large the bid-taker's loss of expected revenue is asymptotically inversely proportional to the square of the number of allowed bid levels.

The model in the following section assumes this asymptotic result as well as equal bid increments and a simple model of auction duration as a function of bid increment. We derive the choice of bid increment that optimizes the trade-off between bid-taker revenue and costs proportional to auction duration. The optimal increment is given by a cube root rule which is relatively insensitive to input parameters. We also note that there may be a difference between the bid increment that maximizes the auctioneer's (short run) expected revenue and the socially optimal one.

Finally, we consider conditions under which it is optimal for bidders faced with finite bid increments to jump over allowed increments. We give a condition under which this is optimal and others under which it is not. We present a two-bidder game in which it is equilibrium behavior for both bidders to avoid jumps.

Oral auctions are complicated phenomena. Their analysis can involve issues related to tradition, sociology and long-run economic consequences as well as those of short-run economics. As we noted above, we are acutely aware of the complicated nature of actual auctions. Our models are not put forward naively as realistic models of oral auctions, but in the hope that they may help in analyzing some of the issues involved in understanding particular auctions and provoking analysis of others.

Appendix

Consider a bidder who has a valuation of $J^{-\alpha}$ ($\alpha > 0$) for an item being sold in an English auction and who has just heard a competitor bid $J-2$. Suppose that given this bid, allowed bids are $J-1, J, J+m$ (where $m > \alpha$), and higher bids. In this Appendix, we analyze this bidder's choice between a final bid of $J-1$ and one of $J$ in order to prove that the former final bid will always maximize the bidder's expected gain if $m \leq 2^{1/2}$ and if the bidder assesses a probability density on the highest competitive valuation given the bid of $J-2$ that is nonincreasing on the range $[J-2, J+m]$.

If we denote the cumulative of this probability assessment by $F(V)$, then its nonincreasing derivative implies that

$$\frac{F(J) - F(J-2)}{2} \geq \frac{F(J+m) - F(J)}{m}.$$ 

Since $F(J-2) = 0$, we have

$$F(J) \geq \frac{(2/m)[F(m+J) - F(J)]}{F(m+J) - F(J)}.$$ 

(A.1)

If the bidder makes a final bid of $J-1$, his expected profit is $(1+\alpha)F(J)$. If he makes a final bid of $J$, his expected profit is $\alpha F(m+J)$. The extra profit, $E$, of skipping the bid to $J$ is given by

$$E = \alpha F(m+J) - (1+\alpha)F(J) = \alpha [F(m+J) - F(J)] - F(J).$$ 

(A.2)

Substituting (A.1) into (A.2) gives

$$E \leq \frac{\alpha - (2/m)}{[F(m+J) - F(J)],}$$

which cannot ever be positive unless $m > 2/\alpha$, or equivalently, $m > 2^{1/2}$.

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of $J-1$, his bid to $J$ is given by $F(m+1)$. The final bid to $J$ is given by $F(J)$.

(1.1)

(1.2)

References


