Two Models of Bid-Taker Cheating in Vickrey Auctions

Michael H. Rothkopf; Ronald M. Harstad


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I. Introduction

In a seminal paper, Vickrey (1961) proposed and analyzed a novel form of auction that is now sometimes named after him. In these "second-price" or "Vickrey" auctions, bidders submit sealed bids, and the bidder with the highest bid wins what is being sold. However, the winner's payment is set, not by the amount of the winner's bid as in standard sealed bidding, but by the amount of the best losing bid, the "second price." For the situation in which the auctioned asset's value to a bidder does not depend on the private information of rival bidders, Vickrey showed that it is a dominant strategy in a second-price auction to submit a "truthfully revealing" bid, that is, a bid exactly equal to the bidder's value for the asset. Furthermore, he pointed out that if bidders followed the dominant strategy, the auction attains an efficient outcome, in that the winner will always be that bidder who values the asset most highly. Finally, Vickrey showed that the seller's expected revenue is the same in a second-price auction as in a standard ("first-price") sealed-bid auction when values are independently drawn.

Some of the Vickrey auction's attractive prop-

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erties have been found to be more robust than his paper suggests. The characterization of a bidder’s best response to rivals’ strategies in a Vickrey auction as bidding that price at which he or she is indifferent between winning and losing is quite general (Levin and Harstad 1986; Bikhchandani and Riley 1991; Harstad 1991). Hence, Vickrey auctions attain an efficient outcome in a wider set of environments than standard sealed-bid auctions. Finally, in a rather general model of symmetric bidding by risk-neutral competitors. Milgrom and Weber (1982) find an expected revenue advantage for a Vickrey auction relative to standard sealed bidding.

For all their advantages, Vickrey auctions are almost unseen outside of financial markets and even in auctions of financial instruments are far closer to being the exception than the rule.\(^1\) By comparison, first-price rules are nearly ubiquitous for sealed-bid auctions of financial instruments, nonfinancial assets, and procurements.

Rothkopf, Teisberg, and Kahn (1990) commented on the rarity of Vickrey auctions, offering two plausible explanations and modeling one of these. In their model, third parties capture a fraction of the economic rent transparently revealed by the use of the Vickrey procedure; in equilibrium, on the average, all of this rent capture is passed on to the bid taker.\(^2\) The second explanation they proposed is concern about cheating, especially bidders’ fear of bid-taker cheating, an issue they raised but did not model.

The importance to the characterizations of Vickrey auctions cited of the presumption that bidders view the bid taker as fully honest can be determined only by modeling explicitly possible bidder suspicions of bid-taker cheating. This need not yield exceptionally surprising conclusions to be a useful modeling step. Engelbrecht-Wiggans (1990)

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1. Citicorp’s weekly commercial paper auctions are run under Vickrey rules, as are the 49-day auctions of variable dividend rate preferred paper. (The latter may be run via Vickrey auction due to tax treatment rather than efficiency considerations.) So-called Dutch repurchase auctions of common stock are more akin to Vickrey auctions (though a large tender can be “pivotal”; see Bagwell 1990) than to the open descending auction used in the flower markets of Holland. There are also a number of isolated occurrences of Vickrey auctions of financial instruments. These include seven sales of Treasury bonds in 1973–74 (Tsao and Vignola 1980), preferred stock offerings of the Tucson Electric Power Company (Tucson Electric Power Company 1986), and various financial issues at Exxon over the last dozen years. In September 1992, the U.S. Treasury Department began experimenting with a multiunit analogue of Vickrey rules for auctioning 2-year and 5-year notes. We have listed all instances we are aware of but make no claim to this list being exhaustive.

2. Engelbrecht-Wiggans and Kahn (1991) also have a model showing an undesirable effect of a subsequent related negotiation on bidding strategies in Vickrey auctions. Motivated by the Rothkopf, Teisberg, and Kahn paper, Nurmi and Salomaa (1993) have proposed a cryptographic version of the Vickrey auction. Several filings commenting on the U.S. Federal Trade Commission’s proposed rule for auctioning the rights to use radio spectrum have indicated that the fact that progressive auctions do not reveal willingness to pay gives them an advantage over Vickrey rules. (See McMillan [1994] for a discussion of the design of this auction.)
noted the lack of models and argued, using assumptions that we contrast below with ours, that if bidders accurately anticipate the extent of cheating, the bid taker will be as well, or in some contexts, even better off with a Vickrey auction. Hence, he questioned the conclusion that fear of bid-taker cheating contributes to the rarity of Vickrey auctions. This article presents two simple but formal models in which the possibility of cheating can make the Vickrey auction an unattractive or unusable alternative to standard sealed bidding and can help explain its rarity.

The next section of this article describes a simple game-theoretic model in which a bid taker's type affects the probability that he will choose to cheat in Vickrey auctions. The bid taker starts the process by deciding whether to hold standard sealed-bid auctions or Vickrey auctions. The bidders' equilibrium behavior creates positive incentives for all bid takers, except the type most prone to cheat, to choose standard sealed-bid auctions, and even sellers of this type are indifferent. A formal mathematical statement of the model is given in Appendix A.

The following section describes a reputation model of repeated Vickrey auctions. The reputation model assumes the seller to be completely rational, but not so the bidders. Until the seller is detected cheating, bidders bid as if he is always honest; afterward, as if he is likely to cheat. This departure from fully rational bidders dramatically simplifies the presentation and is, in many ways, a conservative assumption for the limited conclusions we wish to draw from the model: a seller with probabilistic opportunities to cheat, and finite abilities to resist cheating, will cheat and be caught in finite time and thereafter have no reason to conduct Vickrey auctions. The mathematics for this model is relegated to Appendix B.

Our conclusions are preceded by a short section that discusses why these models do not apply to oral auctions, which in fact are widely used.

II. A Static, Game-Theoretic Model of Cheating in Vickrey Auctions

The basic logic of this model can be made clear without formalism. A seller of an indivisible asset publicly announces whether he is going to conduct a Vickrey auction or a first-price auction. The seller has private information (his "type") which is the probability that he will have and use the opportunity, in a Vickrey auction, of inserting his own fictitious bid into the auction after observing the legitimately submitted bids. Such a fictitious bid would be just below the highest submitted bid and would thus cost the winning bidder the bulk of the difference between his highest bid and the second-highest legitimate bid. Thus, an announced Vickrey auction is in fact run under Vickrey rules with
some probability known to the seller and is, effectively, a first-price auction otherwise. Rational bidders in such an auction, knowing the distribution of types but not this seller's type, will shade their bids in accordance with their beliefs on the probability of the bid taker cheating. If a seller's own probability of cheating is lower than the assumed probability incorporated into the bidders' equilibrium bid functions, it is to his advantage to use a first-price auction rather than a Vickrey auction. (Myerson's [1981] Revenue Equivalence Theorem equates the average revenue in the first-price auction to the average revenue in the supposed Vickrey auction where cheating occurs with the frequency believed by the bidders. Since cheating generates more revenue than not cheating, sellers who have a lower-than-average propensity to cheat will be worse off relative to a first-price auction.) Hence, sellers with low propensities to cheat will choose first-price auctions, and, in equilibrium, bidders will anticipate this. It follows that in equilibrium all seller types except the type with the highest possible cheating probability (who is indifferent) strictly prefer first-price auctions to Vickrey auctions.

Appendix A presents a mathematical version of a slight generalization of this model.

III. A Dynamic Bid-Taker Reputation Model

This section considers a completely different model. In each time period \( t = 1, 2, \ldots \), a seller auctions an indivisible asset. His payoff is the discounted sum of the infinite stream of equilibrium expected revenues in these auctions. Each period, the seller decides whether the auction is run by Vickrey rules or by standard sealed-bid rules.

The seller is not completely averse, in principle, to cheating in Vickrey auctions. There is a chance he will cheat when it is possible and sufficiently advantageous to do so. In addition to the second-highest bid (his honest revenue), cheating brings the immediate gain of a fraction of the difference between the highest and second-highest bids. However, for any auction in which he cheats there is an independent probability that his cheating will be discovered. If it is discovered, we assume that this becomes known to the bidders and thenceforth affects the seller's reputation. Discovery brings a nonnegative penalty as well as the forfeiture equal in present value to the gain from cheating.

3. Note that we are making a modeling choice that contrasts with that of Engelbrecht-Wiggans (1990). He treats the auction form as exogenously fixed and cheating as an exogenously determined uncertain event, and he concludes that bid takers may gain from using Vickrey auctions. If such a conclusion is being considered, it makes sense to build an explicit model that treats bidders as potentially inferring information about bid takers' proclivities from their decisions on auction rules and that allows bidders to suspect that cheating may be an endogenous decision. Doing so changes the conclusions.
This model assumes that bidders’ asset valuations are nontrivially affiliated: a higher valuation by one bidder makes higher valuations by his rivals more likely. Hence, expected revenue in a first-price auction is less than that of a Vickrey auction for which bidders assume that sellers will not cheat. Bidders who have not discovered cheating by the seller bid as if he absolutely will not cheat. However, if he has been discovered cheating even once, bidders in future Vickrey auctions make a sufficient allowance for the possibility of cheating that these auctions no longer yield as much expected revenue as standard sealed bidding.\textsuperscript{4} Except for the assumptions bidders make about whether the seller cheats, bidders are modeled as completely rational. The assumption that bidders do not anticipate cheating until it is discovered is a conservative one for our purposes: it lowers the incentive of a bid taker to cheat. However, unless bidders have a separate disutility of being cheated or utility for punishing cheating, the reaction to the discovery of cheating we assume may be beyond that called for by rationality in this isolated context.\textsuperscript{5}

Appendix B develops the mathematics of our model. However, the intuition is simple. Sellers who have not been caught cheating use Vickrey auctions and occasionally find it worthwhile to cheat. Their cheating is eventually discovered. From that point on, it is unprofitable for the sellers to use Vickrey auctions. Thus, this model exhibits a kind of Gresham’s Law of auction form: sealed-bid auctions eventually drive out Vickrey auctions when there is any possibility of cheating. In the model, having bidders post a bond that is forfeited if they are caught cheating changes the rate at which the process occurs, but not the long-run result.

IV. English Auctions Too?

Traditionally, game-theoretic models treat oral ascending ("English") auctions as quite similar to Vickrey auctions and as completely isomorphic, assuming bidders face no residual interdependent asset value uncertainty. (Indeed, working under this assumption, Vickrey originally conceived the second-price auction as a sealed-bid analogue to

\textsuperscript{4} This would certainly result if bidders used the strategies that they would use in standard sealed-bid auctions. Such a strong reaction is more than we require. The strategies we assume are called "trigger strategies" in the literature on repeated games (see, e.g., Friedman 1971).

\textsuperscript{5} This use of a standard sort of trigger strategy is a deliberate modeling choice. It would be nice to have a full, game-theoretic but nonetheless simple model in which bidders took account of the odds that a seller would cheat in the current auction, given his history of cheating to date and the incentives facing him. Of course, such a model would be far from simple, and it is clear that the forces we describe would still be present. In a broader context than a single-seller, single-perishable-asset economy, it is reasonable for bidders to react strongly to proven bid-taker cheating.
the English auction.) The question naturally arises as to whether the models presented here apply to English auctions.

Unlike Vickrey auctions, English auctions are widely used (see Cassady 1967). The use of shills and of "bid running" is widely suspected and occasionally admitted practice in English auctions. In principle, the price continues to rise in an English auction only if there are still at least two bidders actively competing. Shills are agents of the seller who enter insincere bids. "Bid running" refers to the practice of continuing to raise the price at a point where the auctioneer is aware of only one actively competing bidder. The New York branches of artwork auctioneers Christie's and Sotheby's admit to bid running when the current price is less than the secret reserve price, but they deny ever intentionally running the bid beyond the reserve price. It is frequently the case that they "hammer down" an objet d'art as if sold, only to reveal a few days later that there was no buyer at the hammerdown price. (Such eventual revelation is required by New York law.) The "backups" practice regularly encountered in cattle auctions is essentially an institutionalized admission of bid running and does not appear to be limited to prices below reserve prices (see Harstad and Rothkopf 1991).

The incentives a seller has to cheat in a Vickrey auction clearly are present in English auctions. Because bidders are modeled as assuming that they will only have to pay the highest rival's bid, they have incentive to bid up to their maximum willingness to pay. The seller has an incentive to raise further revenue by driving the winning bidder up closer to his maximum willingness to pay. However, an English auction presents an additional difficulty absent in a Vickrey auction: the winning bidder's maximum willingness to pay is and remains his private information. In a Vickrey auction, a seller may have an opportunity to cheat after all bids, including that of the winner, are known to him. In an English auction, running the bid past the point where the second-highest rival quit competing carries a risk that the remaining bidder will be unwilling to continue; if this happens, the asset will go unsold. Hence, we conclude that our models of bid-taker cheating in Vickrey auctions do not apply to English auctions.

V. Conclusions

While theoretically attractive, Vickrey auctions are remarkably rare. Plausible attempts to explain this rarity turn to (i) the explicit revela-

6. Of course, rational bidders correctly anticipating the possibility of such behavior by the seller would make adjustments in their own behavior. Aside from changing his bidding strategy, a bidder concerned about bid running and attaching sufficient importance to keeping the seller from knowing when he becomes the only bidder still competing can hire a confederate to bid according to prearranged or privately signaled instructions.
tion of private information if bidders adopt strategies that would be dominant were the auction analyzed in isolation and (ii) fears of cheating.

Fears of cheating in auctions can go both ways. Bid takers may fear collusive behavior by a subset of bidders; Robinson (1985), von Ungern-Sternberg (1988), and Mailath and Zemsky (1991) argue that stability of such coalitions is appropriately a more serious concern in a single isolated Vickrey auction than for standard sealed bidding.

This article has provided two models in which bidders’ fear of bid-taker cheating may be a serious concern, helping to explain the rarity of Vickrey auctions. The first model was static and employed the strong assumption that bidders’ valuations are independent, but it concluded that all but the most dishonest type of seller have an incentive to avoid Vickrey auctions.

The second model weakens several assumptions of the first and introduces reputation and a dynamic structure. Even if bidders naively trust a seller who so far has not been detected cheating to report the second-highest bid honestly, the model shows that this trust will eventually be destroyed by a rational but unscrupulous seller. For the purposes of this conclusion, the bidders’ naïveté is a conservative assumption; Vickrey auctions will surely be rarer if bidders initially respond to the unproven possibility of cheating. If bidders’ reaction to the detection of cheating is sufficient to deprive sellers of the revenue advantage of a Vickrey auction, then Vickrey auctions will disappear.

Appendix A

The Static, Game-Theoretic Model

This appendix formalizes and generalizes slightly the logic of the model described above in Section II. In this model, we identify a seller’s type with $q$, the probability that he will have and use the opportunity to cheat in a Vickrey auction. The type of $q \in Q$ is the seller’s private information, with the finite set of types $Q$ contained in [0, 1] and the ex ante probabilities $p_q \in (0, 1)$ of type $q$ assumed to be common knowledge. Let $p = \{p_q, q \in Q\}$, and let $q_{\text{max}}$ denote the maximal element of $Q$.

The seller announces whether he will hold a Vickrey or a first-price auction. Then the $n$ bidders observe their private values and select their bids. If the seller has selected a first-price auction, the auction proceeds in the standard sealed-bid first-price auction manner with no opportunity for the bid taker to cheat. Suppose, however, that the seller has announced a Vickrey auction. Then if the seller is of type 0, the auction proceeds according to the rules usually assumed for Vickrey auctions. For any other type of seller, with probability $q$ a fictitious bid is inserted, transferring a positive fraction of the differ-

7. A model with an endogenous number of bidders, in the spirit of Harstad (1990, 1993), could obtain similar results.
ence between the highest and second-highest bids to the seller. We assume that a threshold extra profit \( T_q \geq 0 \) is necessary to cover the costs and risks of this cheating. Avoiding the trivial, \( T_q \) is assumed small enough that it does not completely prevent cheating by types \( q > 0 \). Bidders are assumed to know the nonincreasing function \( \tau \) mapping \( q \) into \( T_q \).

Each bidder’s private value \( v \) is an independent draw from the cumulative distribution function \( G(v) \) with compact support \([v_\ell, v_u]\). Denote the symmetric equilibrium bid function for the first-price auction by \( b_F(v|G) \). Let \( S \) contained in \( Q \) denote the subset of seller types that bidders assume will use a Vickrey auction. In equilibrium, \( S \) must include any seller types that attain a higher expected revenue and exclude any types that attain a lower expected revenue with the supposed Vickrey auction than with the first-price auction. The symmetric equilibrium bid can be represented as a function by \( b_F(v|S, p, \tau, G) \). It is straightforward to verify that \( b_F(v'|\cdot) \) and \( b_F(v|\cdot) \) are increasing in \( v \) and that \( b_F(v_\ell|\cdot) = v_\ell = b_F(v_u|\cdot) \). Thus, the expected profitability of a bidder with value \( v_\ell \) is zero for any announcement the seller makes, and the bidder drawing the highest value is sure to win. Accordingly, the Revenue Equivalence Theorem of Myerson (1981) applies to this model.\(^8\) Since a winning bidder does not know what type of seller is receiving this revenue, it may be clearer to view Myerson’s result as a “payment equivalence” theorem: either auction procedure leads to the same expected payment (across seller types) from the winning bidder.

The expected payment in a first-price auction is

\[
r_F = \int_{v_\ell}^{v_u} b_F(z|G) \, dG^n(z).
\]

Hence, with equilibrium beliefs as to which seller types will hold a Vickrey auction, payment equivalence implies

\[
r_F = \frac{\sum_{q \in S} p_q r(q|S)}{\sum_{q \in S} p_q}, \tag{A1}
\]

where \( r(q|S) \) is the equilibrium expected payment to a type \( q \) seller when the bidders adjust their bids for the belief that the set of seller types holding Vickrey auctions is \( S \). Let \( S_q = \{ q' \in Q | q' \geq q \} \), including \( q \) and all less honest sellers.

Consider any \( q \in Q(q_{max}) \). As any type \( q' > q \) profits from cheating with greater frequency than \( q \), \( r(q'|S_q) > r(q|S_q) \). Hence, \( r(q|S_q) < r_F \), as by (A1) \( r_F \) is a convex combination of \( r(q|S_q) \) and \( r(q'|S_q) \). Thus, an assumption that \( S_q \) is an equilibrium set of seller types holding Vickrey auctions yields the contraction that \( q \) strictly prefers to hold a first-price auction. As a result, the only equilibria have \( S = \{ q_{max} \} \) or \( S = \{ \cdot \} \). No seller except for the most dishonest type is willing to hold a Vickrey auction, and announcing a Vickrey auction reveals the seller to be of that type \( q_{max} \).

\(^8\) Technically, the probability that the winner will pay a price equal to the legitimate second-highest bid is endogenous, as there is for any \( q \) a calculable probability that the threshold \( T_q \) will be exceeded. This endogeneity is fully consistent with the class of auctions Myerson considers in his model.

\(^9\) If \( I \in Q \) and \( T_1 = 0 \), equilibrium bidding is identical across the two announcements.
Appendix B

The Dynamic Model with Bidder Reputation

This appendix states the mathematics of the model described in Section III. In it, a seller conducts an auction in each time period \( t = 1, 2, \ldots \). His payoff is the discounted sum at rate \( r \) per period of the infinite stream of equilibrium expected revenues in these auctions. At each time, \( t \), the seller announces whether the auction is run by Vickrey rules or by standard sealed-bid rules.

In each Vickrey auction, an opportunity to cheat that the seller would exploit if it is large enough arises with probability \( P \in (0, 1) \), independent of all other random variables. In addition to the second-highest bid, cheating brings the immediate gain of a fraction \( \alpha \in (0, 1) \) of the difference between the highest and second-highest bids. However, for any auction in which he cheats there is an independent probability \( d \in (0, 1) \) that this cheating will be discovered. If it is discovered, we assume that this becomes known to the bidders before the next auction and thenceforth affects the seller’s reputation. (The formulas below would be substantially complicated if discovery took longer, but the conclusions would remain.) Discovery brings a penalty with a present value of \( \Pi \geq 0 \) plus forfeiture equal in present value to the gain from cheating. (If we assume no forfeiture of gain on discovery, the argument made below will hold even if \( d = 1 \).)

In each period, \( t \), each bidder’s private information is a new draw from a fixed distribution. Presentation is simplified by treating bidders as symmetric, but their private information is assumed to be nontrivially affiliated. Thus, for this model, by theorem 14 in Milgrom and Weber (1982), expected revenue in a first-price auction is less than in a Vickrey auction for which bidders assume that sellers will not cheat. Let \( A \geq 0 \) be the additional expected revenue the seller obtains in an auction if he holds a Vickrey auction, bidders assume that he does not cheat, and he does not cheat. It does not matter whether the auction environment is private values, common value, or contains aspects of both.\(^{10}\)

Recall the bidders who have never discovered cheating by this seller bid as if he absolutely will not cheat. However, once he has been discovered cheating once, bidders make a sufficient allowance for the possibility of cheating that future Vickrey auctions no longer yield as much expected revenue as standard sealed bidding. Except for their assumptions about whether the seller cheats, bidders are completely rational.

Consider the first auction \( (t = 1) \), and assume a Vickrey auction is selected. If a seller is discovered cheating, his payoff is reduced in present value by \( L = \Pi + A/r \). Let \( D \) be the random variable which equals the difference between the highest and second-highest bids. We assume there is a probability \( \pi > 0 \) that \( D \) exceeds \( Ld/\alpha(1 - d) \) by any fixed amount that is necessary to overcome the seller’s moral concerns about cheating; this is true, for example,

\(^{10}\) Indeed, the conclusion we reach does not even require equilibrium bidding. The sort of behavioral regularity reported in Kagel, Harstad, and Levin (1987) could support this conclusion, in that bidders bid aggressively enough in Vickrey auctions to make them revenue-superior. The analysis holds for any \( A \geq 0 \), however \( A \) is determined.
whenever bids are drawn conditionally, independently from normal, lognormal, gamma, Weibull, or other distributions which are unbounded above.

Since the seller’s expected net gain from cheating in auction 1 is \( \alpha D (1 - d) - dL \), with probability \( \pi \) he cheats if given the opportunity, that is, he cheats with overall probability \( \pi P \), and he is discovered cheating with probability \( d \pi P > 0 \). In period \( t > 1 \), both the gain and loss are discounted by a factor \( (1 + r)^{-t} \), but the probability that the seller cheats, given that he was not discovered cheating during the first \( t - 1 \) periods, is still \( \pi P \). Hence, the probability that after \( T \) periods the seller has never been caught cheating, \((1 - d \pi P)^{T}\), approaches zero as \( T \) becomes large. Once a seller has been discovered cheating, he reverts to standard sealed-bid auctions thereafter.

As an interesting extension, the cost of cheating can be put under the seller’s control. Let the loss due to being discovered cheating in period 1 become \( L = \pi + A/r + B \), where \( B \geq 0 \) is a bond the seller has placed in escrow as a reputation-building device. As before, the loss due to being discovered cheating in a later period is simply \( L \) discounted. For simplicity, presume that the seller buys a consol in amount \( B \), placing it in escrow, with proceeds paid to the seller until he is discovered cheating, at which time the consol is forfeited to his victim. Adding this to the model leads to an increase in the threshold \( D \), and with it a reduction in \( \pi \), as \( B \) is increased. However, \( \pi \) does not fall to zero for any finite \( B \). Thus, the bond serves merely to delay the inevitable. The expected time until discovery increases but is still finite. Indeed, as bidders have been modeled here, \( B = 0 \) would be the seller’s optimal choice, since whenever he does cheat, his expected supergame payoff is thereby increased; he would prefer to have his opportunity to cheat sooner.

Rational modeling of bidders would have them always considering their best estimate of the threshold \( D \) in formulating their bids in each Vickrey auction. In this case, it is possible that setting \( B > 0 \) would be wise for a seller, in order to move bidders closer to the bids they would make in a Vickrey auction with absolutely no chance of cheating, that is, in order to come closer to gaining fully \( A \) when using a Vickrey auction. However, raising \( B \) would decrease the expected frequency of cheating, which is disadvantageous to the seller, especially if \( d < 1 \), so the overall effect would be ambiguous. Of course, the advantage of \( B > 0 \) would still last only until the seller is discovered cheating, which remains arbitrarily likely in finite time.

It is also possible to generalize the results for this model by introducing inflation at a nonnegative rate \( i \) less than the interest rate. The distribution from which values are drawn would be scaled by a factor \( 1 + i \) each period, where \( 0 \leq i < r \). Other economic factors are also so scaled: \( \pi, A, D, \) and \( B \). Results are unchanged, except that \( r \) is replaced by \( r - i \) in all of the formulas above.\(^{11}\)

\(^{11}\) Similarly, it is possible to move away from exact identity of value distributions over time in a stochastic sense. That is, bidders’ values could be drawn affiliated with an underlying trend variable, which might, e.g., react to boom-and-bust cycles in the economy or industry. Details would be substantially complicated, but cheating discovered in finite time with a probability of \( 1 \) would remain.
References


